Math 2415, Quiz #11, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Find the Fourier series for $f(x) = \begin{cases} 0, & -1 \le x < 0 \\ x^2, & 0 \le x \le 1 \end{cases}$, $f(x+2) = f(x)$.

$$a_0 = \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$a_n = \int_0^1 x^2 \cos(n\pi x)\,dx = \frac{x^2}{n\pi}\cancel{\sin(n\pi x)} + \frac{2x}{n^2\pi^2}\cos(n\pi x) - \frac{2}{n^3\pi^3}\cancel{\sin(n\pi x)}\Big|_0^1 = \frac{2}{n^2\pi^2}\cos(n\pi) = \frac{2(-1)^n}{n^2\pi^2}$$

$$b_n = \int_0^1 x^2 \sin(n\pi x)\,dx = -\frac{x^2}{n\pi}\cos(n\pi x) + \frac{2x}{n^2\pi^2}\cancel{\sin(n\pi x)} + \frac{2}{n^3\pi^3}\cos(n\pi x)\Big|_0^1 =$$

$$-\frac{1}{n\pi}(-1)^n + \frac{2}{n^3\pi^3}(-1)^n - \frac{2}{n^3\pi^3} = (-1)^n\left(\frac{2-n^2\pi^2}{n^3\pi^3}\right) - \frac{2}{n^3\pi^3} = \frac{(-1)^n(2-n^2\pi^2)-2}{n^3\pi^3}$$

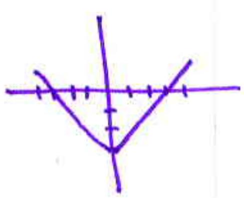
$$f(x) = \frac{1}{6} + \sum_{n=1}^{\infty} \frac{2(-1)^n}{n^2\pi^2}\cos(n\pi x) + \left[\frac{(-1)^n(2-n^2\pi^2)-2}{n^3\pi^3}\right]\sin(n\pi x)$$

2. Extend the given functions as a) an even function, b) an odd function. State the period of the extended function. Sketch the resulting graphs in each case.

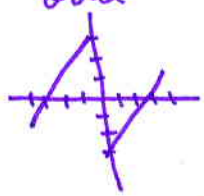
i. $f(x) = x - 3, 0 < x < 4$

period $= 8$

even



odd

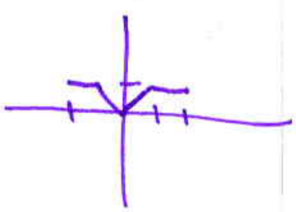


odd $f(x) = \begin{cases} x+3 & -4 \le x < 0 \\ x-3 & 0 \le x < 4 \end{cases}$

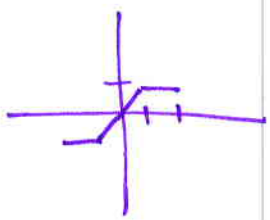
even $f(x) = \begin{cases} -x-3 & -4 \le x < 0 \\ x-3 & 0 \le x < 4 \end{cases}$

ii. $f(x) = \begin{cases} x, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$

even



odd



period=4

odd $f(x) = \begin{cases} -1 & -2 \le x < -1 \\ x & -1 \le x < 1 \\ 1 & 1 \le x < 2 \end{cases}$

even $f(x) = \begin{cases} 1 & -2 \le x < -1 \\ -x & -1 \le x < 0 \\ x & 0 \le x < 1 \\ 1 & 1 \le x < 2 \end{cases}$