Math 2415, Quiz #1, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Perform the indicated operations on the matrices/vectors given that $A = \begin{bmatrix} 2 & -1 \\ 5 & 4 \end{bmatrix}, B = \begin{bmatrix} 4 & -2 \\ -3 & 7 \end{bmatrix}, \vec{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \vec{y} = \begin{bmatrix} -9 \\ 4 \end{bmatrix}$.

   a. $2A^T$

   $A^T = \begin{bmatrix} 2 & 5 \\ -1 & 4 \end{bmatrix}$ $\quad 2A^T = \begin{bmatrix} 4 & 10 \\ -2 & 8 \end{bmatrix}$

   b. $\vec{x} - 4\vec{y}$

   $\begin{bmatrix} 3 \\ 1 \end{bmatrix} - 4\begin{bmatrix} -9 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 36 \\ -16 \end{bmatrix} = \begin{bmatrix} 39 \\ -15 \end{bmatrix}$

   c. $(A - 3B^T)^T$

   $B^T = \begin{bmatrix} 4 & -3 \\ -2 & 7 \end{bmatrix}$ $\quad -3B^T = \begin{bmatrix} -12 & +9 \\ 6 & -21 \end{bmatrix}$ $\quad A - 3B^T = \begin{bmatrix} -10 & 8 \\ 11 & -17 \end{bmatrix}$

   $(A-3B^T)^T = \begin{bmatrix} -10 & 11 \\ 8 & -17 \end{bmatrix}$

2. Use the definitions of the hyperbolic trig functions to prove that $\cosh^2 x - \sinh^2 x = 1$.

$$\left(\frac{e^x+e^{-x}}{2}\right)^2 - \left(\frac{e^x-e^{-x}}{2}\right)^2 = \frac{1}{4}\left[\left(e^{2x}+2+e^{-2x}\right)-\left(e^{2x}-2+e^{-2x}\right)\right]$$

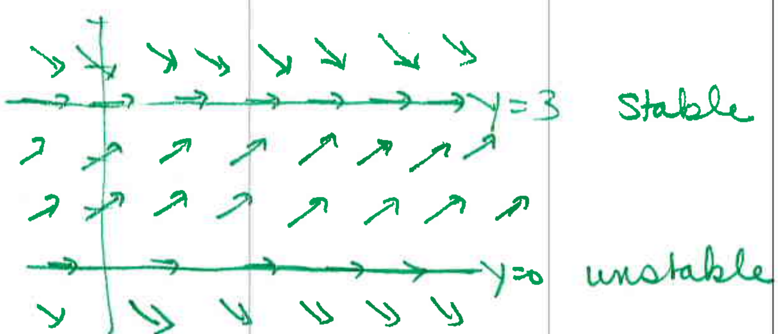
$$= \frac{1}{4}\left[\left(e^{2x}+2+e^{-2x}-e^{2x}+2-e^{-2x}\right)\right] = \frac{1}{4}(4) = 1$$

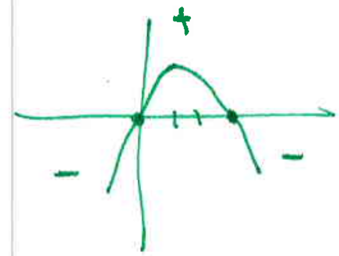
3. Write the complex number e^{3-5i} in standard form using Euler's formula.

$$e^{3-5i} = e^{3} \cdot e^{-5i} = e^{3}\left(\cos(-5) + i\sin(-5)\right) =$$

$$\left[e^{3}\cos(5)\right] + \left[-e^{3}\sin(5)\right] i$$

4. Plot the direction field for the autonomous equation $y' = y(3-y)$, along with the phase portrait.





5. For the direction field plotted below, graph three trajectories from three different initial conditions: (-2,1), (-1,-2), (1,-2).