Math 2415, Quiz #5, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Reduce the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 0 & 5 \end{bmatrix}$ to reduced echelon form and solve the system it represents.

$\begin{bmatrix} 1 & 1/2 & -1/2 \\ 3 & 0 & 5 \end{bmatrix}$ $-3R_1 + R_2 \to R_2$

$-3 \quad -3/2 \quad 3/2$

$\begin{bmatrix} 1 & 1/2 & -1/2 \\ 0 & -3/2 & 13/2 \end{bmatrix}$ $-2/3 R_2 \to R_2$

$\begin{bmatrix} 1 & 1/2 & -1/2 \\ 0 & 1 & -13/3 \end{bmatrix}$ $-1/2 R_2 + R_1 \to R_1$

$0 \quad -1/2 \quad 13/6$

$\begin{bmatrix} 1 & 0 & 5/3 \\ 0 & 1 & -13/3 \end{bmatrix}$ $x_1 = 5/3$, $x_2 = -13/3$ $\vec{x} = \begin{bmatrix} 5/3 \\ -13/3 \end{bmatrix}$

2. Solve the exact equation $\left(\frac{y}{x} + 6x\right) dx + (\ln x - 2) dy = 0, y > 0.$

M N

$\frac{\partial N}{\partial x} = \frac{1}{x}$ $\frac{\partial M}{\partial y} = \frac{1}{x}$ ✓

$\int \frac{y}{x} + 6x \, dx = y \ln x + 3x^2 + f(y)$

$\int \ln x - 2 \, dy = y \ln x - 2y + g(x)$

$\varphi(x, y) = y \ln x + 3x^2 - 2y + C = 0$

3. Solve the initial value problem $2y'' + y' - 3y = 0, y(0) = 4, y'(0) = 1$.

$$2r^2 + r - 3 = 0$$
$$(2r+3)(r-1) = 0$$
$$r = -3/2 \quad r = 1$$

$$Y = C_1 e^{-3/2 t} + C_2 e^{t} \qquad y' = -3/2\, C_1 e^{-3/2 t} + C_2 e^{t}$$

$$4 = C_1 + C_2 \qquad 1 = -3/2\, C_1 + C_2$$
$$-1 = 3/2\, C_1 - C_2$$

$$3 = \frac{5}{2} C_1 \qquad 1 + \frac{3}{2}\left(\frac{6}{5}\right) = C_2$$
$$\frac{6}{5} = C_1 \qquad 1 + \frac{9}{5} = \frac{14}{5} = C_2 \qquad \Rightarrow\ y(t) = \frac{6}{5} e^{-3/2 t} + \frac{14}{5} e^{t}$$

4. Solve the boundary value problem $y'' - 2y = 0, y(0) = 3, y(1) = 4$. Is the solution unique?

yes

$$r^2 - 2 = 0$$
$$r = \pm\sqrt{2}$$

$$Y = C_1 e^{-\sqrt{2}t} + C_2 e^{\sqrt{2}t}$$

$$3 = C_1 + C_2 \qquad C_1 = 3 - C_2$$
$$\left(4 = C_1 e^{-\sqrt{2}} + C_2 e^{\sqrt{2}}\right) e^{\sqrt{2}}$$

$$4e^{\sqrt{2}} = C_1 + C_2 e^{2\sqrt{2}} \qquad Y = \left(\frac{3e^{2\sqrt{2}} - 4e^{\sqrt{2}}}{e^{2\sqrt{2}} - 1}\right) e^{-\sqrt{2}t} + \left(\frac{4e^{\sqrt{2}} - 3}{e^{2\sqrt{2}} - 1}\right) e^{\sqrt{2}t}$$
$$-3 = -C_1 - C_2$$

$$4e^{\sqrt{2}} - 3 = C_2\left(e^{2\sqrt{2}} - 1\right)$$

$$C_2 = \frac{4e^{\sqrt{2}} - 3}{e^{2\sqrt{2}} - 1}$$

$$C_1 = 3 - \frac{4e^{\sqrt{2}} - 3}{e^{2\sqrt{2}} - 1} = \frac{3e^{2\sqrt{2}} - 3 - 4e^{\sqrt{2}} + 3}{e^{2\sqrt{2}} - 1} = \frac{3e^{2\sqrt{2}} - 4e^{\sqrt{2}}}{e^{2\sqrt{2}} - 1}$$