Math 2415, Quiz #6, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Determine if the functions $y_1 = e^{2t}, y_2 = te^{2t}$ form a fundamental set by finding the value of the Wronskian.

$$W = \begin{bmatrix} e^{2t} & te^{2t} \\ 2e^{2t} & e^{2t} + 2te^{2t} \end{bmatrix} = e^{4t} + 2te^{4t} - 2te^{4t} = e^{4t} \neq 0$$

These form a fundamental set

2. For the second order differential equation $t(t-4)y'' + 3ty' + 4y = 2$, use Abel's Theorem to find the value of the Wronskian.

$$y'' + \frac{3t}{t(t-4)}y' + \frac{4y}{t(t-4)} = \frac{2}{t(t-4)}$$

$$W = ce^{-\int \frac{3}{t-4}\,dt} = ce^{-3\ln|t-4|} = c\left[\frac{1}{(t-4)^3}\right] \neq 0 \text{ unless } c = 0$$

3. Find the determinant of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 1 \\ 4 & -2 & 0 \end{bmatrix}$.

$$1\begin{vmatrix} 3 & 1 \\ -2 & 0 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix} + (-1)\begin{vmatrix} 2 & 3 \\ 4 & -2 \end{vmatrix} =$$

$$2 - (-4-12) = 2 + 16 = 18$$

4. Find the solutions to the differential equation $y'' + 6y' + 13 = 0$.

$$r^2 + 6r + 13 = 0$$

$$\frac{-6 \pm \sqrt{36 - 52}}{2} = \frac{-6 \pm \sqrt{-16}}{2} = \frac{-6 \pm 4i}{2} = -3 \pm 2i$$

$$y_1(t) = c_1 e^{-3t}\cos 2t \qquad y_2(t) = c_2 e^{-3t}\sin 2t$$