MATH 1030 Course Review

***Directions**: Some of the questions on this review may require the use of the graphing calculator; others may require you to show all work. If an algebraic answer is required and work is not shown, you may not receive full credit on the final exam. On the final exam you must show work in the spaces provided and show graphs on the grids provided. Partial credit may be awarded on most problems. Reduce fractions to lowest terms. The final exam counts as 25% of your overall grade and contains 200 possible points. You will have 1 hour and 50 minutes to complete the final exam, but this review will most likely take you at least twice as long to complete.*

1. Use the concept of slope to explain the difference between parallel and perpendicular lines.

2. $(2, 3)$ and $(5, -2)$ are on the same line. $(-1, -6)$ and (A, B) are on a different line of their own. These two lines are perpendicular. Determine possible values for A and B.

3. Approximate solutions to the system of equations $\begin{cases} 6x - y = -5 \\ 4x - 2y = 6 \end{cases}$ by the graphing method. Include a "nice" graph. A "nice" graph includes important points, useful points, axes with a scale, and omits extraneous information.

4. Use your calculator to approximate the solution to the systems of equations. Express your answer rounded to two decimal places.

   a) $\begin{cases} y = -1.26x - 16.43 \\ y = 5.61x + 3.65 \end{cases}$ b. $\begin{cases} y = \frac{1}{-1.2}x - 34.7 \\ y = -\pi\, x + 3.65 \end{cases}$

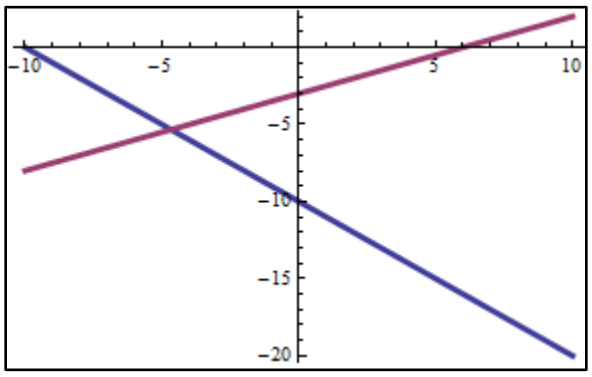
5. Algebraically solve the system of equations below using both the substitution and the elimination methods.

   a) $\begin{cases} \frac{6}{5}x - y = -5 \\ 4x - \frac{2}{3}y = 3 \end{cases}$ b) $\begin{cases} 6x - \frac{2}{5}y = -5 \\ 4x - \frac{3}{4}y = \frac{3}{5} \end{cases}$ c) $\begin{cases} 6x - y = \frac{-5}{4} \\ 4x - 2^2 y = 6 \end{cases}$ d) $\begin{cases} 6x - y = -5 \\ 4x - (-2)y = 6 \end{cases}$

6. Is the ordered pair $(-5, 6)$ a solution of the system: $\begin{cases} \frac{x}{15} + \frac{1}{3}y = 1 \\ 2x - \frac{y}{19} = 13 \end{cases}$? Explain

   [Hint: You do not need to solve the system. Look at the signs in the second equation.]

7. Below is a graph of the lines determining this system of linear equations: $\begin{cases} Equation\ \#1 \\ Equation\ \#2 \end{cases}$



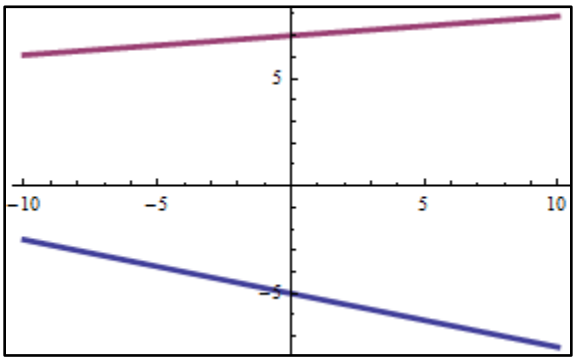
Could the ordered pair $(-15, 6)$ be a solution of the system? Explain your Thinking

8. Describe the three possible graphical representations of a linear system of two equations in two variables, and specify the number of solutions contained in each graphical representation.

9. How many solutions can a system of two linear equations have? Explain

10. Plot the point $(3, -4)$ on a coordinate plane. Draw two different lines that intersect at this point. Your picture represents a system of linear equations. Write the out the system in equation form.

11. Below is a graph of the lines determining this system of linear equations: $\begin{cases} Equation\ \#1 \\ Equation\ \#2 \end{cases}$



Which of the following are true?

a) $(-11, 1)$ seems like a reasonable solution to the system.
b) This system has no solutions.
c) The solution to this system would be represented by a point in the second quadrant.
d) It is impossible for this system to have an infinite number of solutions.
e) $(6, -5)$ is a good approximation to the solution to this system.

12. If $\begin{cases} Equation\ \#1 \\ Equation\ \#2 \end{cases}$ is a system of linear equations that has exactly one solution, then what can you say about the slopes of the two lines corresponding to these two equations?

13. If $\begin{cases} Equation\ \#1 \\ Equation\ \#2 \end{cases}$ is a system of linear equations that has no solutions, then what can you say about the slopes of the two lines corresponding to these two equations?

Solve problems 14-17 using a system of equations. Define descriptive and useful variables and write a system of equations that could be used to solve the problem, solve, and state the solutions precisely.

14. For this problem, define each variable used and write a system of equations that would describe the situation. **DO NOT SOLVE THE SYSTEM…JUST SET IT UP**

    A chemist needs 210 milliliters of a 53% alcohol solution, but has only 41% and 72% alcohol solutions available. How many milliliters of each should be mixed to get the desired alcohol solution?

    a) Let _____ represent _____
    b) Let _____ represent _____
    c) System of Equations _____
    _____

15. James has available a 10% alcohol solution and a 60% alcohol solution. Find how many liters of each solution he should mix to make 50 liters of a 40% alcohol solution.

16. A hardware store stocks two types of hammers. The store has a total of 42 hammers, with sledge hammers selling at $22.95 each and claw hammers selling at $10.95 each. If the total value of the hammers is $639.90, how many of each type are in stock?

17. Hertz car rental agency charges $50 daily plus 10 cents per mile. Budget charges $20 daily plus 25 cents per mile. Find the daily mileage for which the Budget charge for the day is the same as that of the Hertz charge for the day.

18. Graph the solution set for the system $\begin{cases} 6x - y \leq -5 \\ 4x - 2y > 6 \end{cases}$

19. Graph the solution set for the system $\begin{cases} 2A - B \leq -4 \\ 4A - 3B > 7 \end{cases}$

20. One vs. two variables
    a) If x is the only variable involved in the situation, then how would you graph $x \leq 5$?
    b) If x and y are two variables involved in the situation, then how would you graph $x \leq 5$?

21. Use exponent properties to represent the following expressions using only positive exponents:

    a) $5w^0$ b) $(3xy^{-3})(-x^{-3}y^6)$ c) $\frac{9a^{-3}}{6} \cdot \frac{4}{a^2}$ d) $\frac{m}{s^2} \cdot s \cdot \frac{k}{s\,m}$ e) $\left(\frac{h^{-3}}{h^{-5}}\right)^{-2}$

22. What is the degree of this polynomial? $(-345 + y^2 - 52y)y^2 - (158y^3 + 45y - 871y^2 + 95)$

23. Write each polynomial as a sum of monomial terms (multiply out and collect like terms):

a) $(5x+3y)(x-2y)$ b) $(x-1)(x^2+x+1)$ c) $(5x-4y)^2$ d) $5x-6(4x+3)$ e) $(2m-3n)(2m+3n)$

24. Divide $15x^3 - 5x^2 + 10x$ by $5x^2$ using long division.

25. Reduce $\frac{15x^3-5x^2+10x}{5x^2}$, but leaving it in fraction form.

26. Divide $21x^2 + x - 10$ by $3x - 2$ using long division.

27. Reduce $\frac{21x^2+x-10}{3x-2}$, but leaving it in fraction form.

28. Divide $2x^3 + 3x - 4$ by $x + 2$ using long division.

29. Reduce $\frac{2x^3+3x-4}{x+2}$, but leaving it in fraction form.

30. When the calculator displays the result 3.234E–35, what number is being indicated?
(i.e., how would you write the number on paper without listing all the zeros?)

31. Write 31,000,000,000 in scientific notation.

32. Write 3.1×10^{-4} in standard notation, without exponents.

33. Write the following expression in scientific notation. (Use a calculator to check the result.)

$$\frac{(2 \times 10^3)(3 \times 10^8)}{(8 \times 10^4)}$$

34. Factor completely:

a) $3y^2 - 8y - 3$ b) $2m^4 - 72m^2$ c) $k^3 - 8$ d) $x^2 - x - 42$

35. Write each polynomial as a product:

a) $3b + bx + 3y + xy$ b) $2A(A - B^2) - 5A(A - B^2)$ c) $9w^4 + 36w^2$ d) $49T^2 - 36$

36. Factor each polynomial:

a) $36G^2 - 60G + 25$ b) $27x^3 + 125y^3$

37. Factor out $h - 2$ from $h^3 + 4h^2 - 15h + 6$

38. Factor $3x^2y$ out from $3x^6y^5 + (-6)x^5y^4 + 15x^5y^3 - 18x^4y$

39. Factor $x^{\frac{1}{2}}$ from $x^3 + 4x^2 - 15x^{\frac{3}{2}} + 6x^{\frac{1}{2}}$

40. Show that -1 is not a solution of $3x^7 - x^5 + 2x^4 + 5x^2 = 13x - 2$

41. Determine a value for k, so that -1 is a solution of $3x^7 - x^5 + 2x^4 + 5x^2 = 13x + k$

42. Solve each equation algebraically:

a) $6y^2 - 15y = 0$ b) $(3A + 1)(2A - 5) = 39$ c) $6t^2 - 5t - 4 = 0$ d) $T^3 = 9T$

43. Determine all solutions of $3t^2 + (-5)t + 4 = 2t^2 - 4t + 10$

44. Describe the solution set of $\frac{t^2}{6} - \frac{t}{6} = 1$

45. An object is dropped off the top of a building that is 256 feet high. The distance, in feet, above the ground at s seconds is given by $H(s) = -16s^2 + 256$. Determine when the object hits the ground. Solve this problem both algebraically and using your calculator.

46. The shorter leg of a right triangle is 3 centimeters less than the other leg. Find the length of the two legs if the hypotenuse is 15 centimeters.

47. True or False
If you multiply two prime polynomials together, then the resulting polynomial is also prime. Explain.

48. True or False
If T and W are two polynomials, then the degree of their product $T \cdot W$ is the sum of their individual degrees

degree of $T \cdot W$ = degree of T + degree of W