Math 2153, Quiz #11, Summer 2015 Name KEY

Instructions: Show all work. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Find a vector normal to the surface $\vec{r}(u,v) = 2u\cos(v)\hat{\imath} + 2u\sin(v)\hat{\jmath} + \frac{1}{2}u^2\hat{k}$.

$\vec{r}_u = 2\cos v\,\hat{\imath} + 2\sin v\,\hat{\jmath} + u\hat{k}$

$\vec{r}_v = -2u\sin v\,\hat{\imath} + 2u\cos v\,\hat{\jmath} + 0\hat{k}$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2\cos v & 2\sin v & u \\ -2u\sin v & 2u\cos v & 0 \end{vmatrix} = (0 - 2u^2\cos v)\hat{\imath} - (0 + 2u^2\sin v)\hat{\jmath} + (4u\cos^2 v + 4u\sin^2 v)\hat{k}$$

$$= -2u^2\cos v\,\hat{\imath} - 2u^2\sin v\,\hat{\jmath} + 4u\hat{k} = \vec{n}$$

2. Find the curvature of the vector-valued function $\vec{r}(t) = 4\cos t\,\hat{\imath} + \sin t\,\hat{\jmath} + 2t\hat{k}$ at the point $t = \pi$.

$$k = \frac{\|\vec{v} \times \vec{a}\|}{\|\vec{v}\|^3} = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$$

$r'(t) = -4\sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + 2\hat{k}$

$r''(t) = -4\cos t\,\hat{\imath} - \sin t\,\hat{\jmath} + 0\hat{k}$

$\|r'(t)\| = \sqrt{16\sin^2 t + \cos^2 t + 4} = \sqrt{15\sin^2 t + 1 + 4} = \sqrt{15\sin^2 t + 5}$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4\sin t & \cos t & 2 \\ -4\cos t & -\sin t & 0 \end{vmatrix} = (0 - 2\sin t)\hat{\imath} - (0 + 8\cos t)\hat{\jmath} + (4\sin^2 t + 4\cos^2 t)\hat{k}$$

$$= (-2\sin t)\hat{\imath} - 8\cos t\,\hat{\jmath} + 4\hat{k}$$

$\|r'(t) \times r''(t)\| = \sqrt{4\sin^2 t + 64\cos^2 t + 16} = \sqrt{4 + 60\cos^2 t + 16} = \sqrt{20 + 60\cos^2 t}$

$$k = \frac{\sqrt{20 + 60\cos^2 t}}{(15\sin^2 t + 5)^{3/2}} \quad \text{at } t = \pi \quad \frac{\sqrt{20 + 60(1)}}{(15(0) + 5)^{3/2}} = \frac{\sqrt{80}}{\sqrt{125}} = \sqrt{\frac{80}{125}} = \sqrt{\frac{16}{25}} = \boxed{\frac{4}{5}}$$