Instructions: Show all work. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Calculate the flow through the closed surface $\vec{F}(x, y, z) = (x + y)\hat{\imath} + y\hat{\jmath} + z\hat{k}, S: z = 16 - x^2 - y^2, z = 0$ using the Divergence Theorem.

$$\vec{\nabla} \cdot \vec{F} = 1 + 1 + 1 = 3$$

$$z = 16 - r^2$$

$$\iiint_V 3\, dV = \int_0^{2\pi} \int_0^4 \int_0^{16-r^2} 3r\, dz\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^4 3r(16 - r^2)\, dr\, d\theta = \int_0^{2\pi} \int_0^4 48r - 3r^3\, dr\, d\theta =$$

$$\int_0^{2\pi} 24r^2 - \frac{3}{4}r^4 \Big|_0^4 d\theta = \int_0^{2\pi} 384 - \frac{192}{} \, d\theta = 192\int_0^{2\pi} d\theta = \boxed{384\pi}$$

2. Calculate the flow through the closed surface $\vec{F}(x, y, z) = 2x\hat{\imath} - 2y\hat{\jmath} + z^2\hat{k}$, $S: cylinder \ x^2 + y^2 = 4, 0 \le z \le 5$ using the Divergence Theorem.

$$r = 2$$

$$\vec{\nabla} \cdot \vec{F} = 2 - 2 + 2z = 2z$$

$$\int_0^{2\pi} \int_0^2 \int_0^5 2z r\, dz\, dr\, d\theta = \int_0^{2\pi} \int_0^2 z^2 \Big|_0^5 r\, dr\, d\theta = \int_0^{2\pi} \int_0^2 25r\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{25}{2}r^2 \Big|_0^2 = \int_0^{2\pi} 50\, d\theta = \boxed{100\pi}$$