

Instructions: Show all work. Give exact answers (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Find the general solution to the second order ODE $y'' + 12y' - 35y = \sin t + 4e^{7t}$.

$$r^2 + 12r - 35 = 0$$

$$r = \frac{-12 \pm \sqrt{144 + 140}}{2} = \frac{-12 \pm \sqrt{284}}{2} = -6 \pm \sqrt{71}$$

$$y(t)_{\text{homogeneous}} = C_1 e^{(-6+\sqrt{71})t} + C_2 e^{(-6-\sqrt{71})t}$$

$$\text{Ansatz: } A \sin t + B \cos t + C e^{7t} = Y(t)$$

$$Y'(t) = A \cos t - B \sin t + 7C e^{7t}$$

$$Y''(t) = -A \sin t - B \cos t + 49C e^{7t}$$

$$-A \sin t - B \cos t + 49C e^{7t} + 12(A \cos t - B \sin t + 7C e^{7t}) - 35(A \sin t + B \cos t + C e^{7t}) =$$

$$\sin t + 4e^{7t}$$

Sines

$$-A - 12B - 35A = 1 \Rightarrow -36A - 12B = 1$$

Cosines

$$-B + 12A - 35B = 0 \Rightarrow -36B + 12A = 0$$

$$\begin{bmatrix} -36 & -12 & | & 1 \\ 12 & -36 & | & 0 \end{bmatrix}$$

$$\Rightarrow A = -\frac{1}{40}$$

$$B = -\frac{1}{120}$$

exp.

$$49C + 84C - 35C = 4 \Rightarrow 98C = 4 \quad C = \frac{2}{49}$$

$$y(t) = C_1 e^{(-6+\sqrt{71})t} + C_2 e^{(-6-\sqrt{71})t} - \frac{1}{40} \sin t - \frac{1}{120} \cos t + \frac{2}{49} e^{7t}$$