MAT 202, Exam #1, Part I, Summer 2016 Name KEY

Instructions: Show all work. You may **not** use a calculator on this portion of the exam. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals). Reduce as much as possible. Be sure to complete all parts of each question. Provide explanations where requested. When you are finished with this portion of exam, get Part II.

1. Determine if each statement is True or False. (1 points each)
   a. T F A system of one linear equation in two variables is always consistent. $X+Y+3=X+Y$.
   b. T F A linear system can have exactly two solutions. none, 1 or ∞ only
   c. T F A 4x7 matrix has four columns. 4 rows
   d. T F Every matrix is row-equivalent to a matrix in echelon form.
   e. T F There is only one way to parametrically represent the solution set of a linear equation.
   f. T F For the product of two matrices to be defined, the number of columns of the first matrix must equal the number of rows of the second matrix.
   g. T F Matrix multiplication is commutative.
   h. T F If the matrices A, B, C satisfy $AB = AC$, then $B = C$. only if A is invertible
   i. T F If A can be row-reduced to the identity, then A is nonsingular.
   j. T F The zero matrix is an elementary matrix. all elementary matrices are invertible
   k. T F The inverse of an elementary matrix is an elementary matrix.
   l. T F Addition of matrices is not commutative. it is Commutative
   m. T F All $n \times n$ matrices are invertible. Some are singular

2. Solve the system of equations $\begin{cases} x + 3y = 2 \\ -x + 2y = 3 \end{cases}$ by writing the system as an augmented matrix and row-reducing by hand. (5 points)

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ -1 & 2 & 3 \end{array}\right] \; R_1+R_2 \to R_2 \; \left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 5 & 5 \end{array}\right] \; 1/5 R_2 \to R_2$$

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 1 & 1 \end{array}\right] -3R_2+R_1 \to R_1 = \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 1 \end{array}\right]$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

3. Solve the system $\begin{cases} x_1 - 2x_2 - 8x_3 = 0 \\ 3x_1 + 2x_2 \qquad\quad = 0 \end{cases}$ and write the solution in parametric form. (5 points)

$$\left[\begin{array}{ccc|c} 1 & -2 & -8 & 0 \\ 3 & 2 & 0 & 0 \end{array}\right] \quad -3R_1+R_2 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & -2 & -8 & 0 \\ 0 & 8 & 24 & 0 \end{array}\right] \quad \tfrac{1}{8}R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -8 & 0 \\ 0 & 1 & 3 & 0 \end{array}\right] \quad 2R_2+R_1 \to R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 0 \end{array}\right]$$

$$\begin{array}{l} X_1 - 2X_3 = 0 \\ X_2 + 3X_3 = 0 \\ \quad X_3 = X_3 \end{array} \Rightarrow \begin{array}{l} X_1 = 2X_3 \\ X_2 = -3X_3 \\ X_3 = X_3 \end{array} \Rightarrow \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} t$$

$(X_3 = t)$

4. Give an example of a 3 × 4 matrix in echelon form whose solution is inconsistent. (3 points)

$$\left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 1 \end{array}\right]$$

5. For the matrices $A = \begin{bmatrix} -4 & 3 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}, C = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}, D = \begin{bmatrix} 3 & 2 & 1 \\ -3 & 0 & 4 \\ 4 & -2 & -4 \end{bmatrix}$.

Calculate the following matrices. If the operation is not defined, explain why not. (4 points each)

a. $2A - 5B^T$

$$\begin{bmatrix} -8 & 6 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 10 & -5 \\ -5 & 40 \end{bmatrix} = \begin{bmatrix} -18 & 11 \\ 9 & -42 \end{bmatrix}$$

b. C^TC

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 4 & 6 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 4+9+1 & 2-12+6 \\ 2-12+6 & 1+16+36 \end{bmatrix} = \begin{bmatrix} 14 & -4 \\ -4 & 53 \end{bmatrix}$$

c. BA

$$\begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}\begin{bmatrix} -4 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -8-2 & 6+1 \\ 4+16 & -3-8 \end{bmatrix} = \begin{bmatrix} -10 & 7 \\ 20 & -11 \end{bmatrix}$$

d. CD

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 3 & 2 & 1 \\ -3 & 0 & 4 \\ 4 & -2 & -4 \end{bmatrix}$$

3×2 3×3

not defined
inside dimensions do not match

e. A^{-1}

$$\frac{1}{4-6}\begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1/2 & 3/2 \\ 1 & 2 \end{bmatrix}$$

6. Consider matrices of the indicated sizes. Describe the size of the matrix product. (2 points each)
   a. $(4 \times 1) \cdot (1 \times 4)$ (4×4)

   b. $(2 \times 2) \cdot (2 \times 3)$ (2×3)

   c. $(m \times n) \cdot (n \times p)$ $(m\times p)$

7. Is it possible for a system of linear equations with fewer equations than variables to have no solution? If so, give an example. If not, explain why not. (3 points)

yes. $\begin{cases} 3x+4y+z=11 \\ 6x+8y+2z=2 \end{cases}$

8. What does it mean for a system to be overdetermined? Give an example. (3 points)

there are more equations than there are variables

the system may or may not be consistent

$$\begin{cases} 2x+y=4 \\ x-y=11 \\ 3x+2y=5 \end{cases}$$

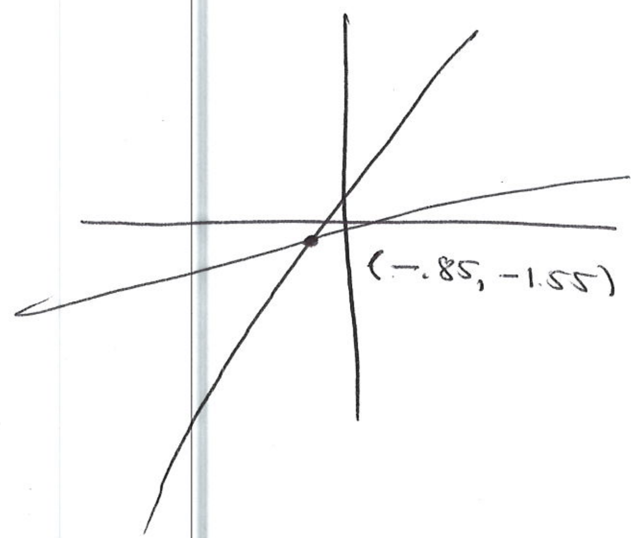
MAT 202, Exam #1, Part II, Summer 2016 Name ______________________

Instructions: Show all work. You **may** use a calculator on this portion of the exam. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question. Provide explanations where requested.

1. Graphically solve the system of equations $\begin{cases} -5.3x + 2.1y = 1.25 \\ 0.8x - 1.6y = 1.8 \end{cases}$. Write the solution in vector form, and round the solutions to 3 decimal places if needed. Classify the system as consistent or inconsistent; classify the solution as independent or dependent. Sketch the graph. (6 points)

$y = \frac{5.3x + 1.25}{2.1}$ $y = \frac{.8x - 1.8}{1.6}$

Consistent
independent



$\begin{bmatrix} -.85 \\ -1.55 \end{bmatrix}$

2. Find a cubic model for the data in the table below.

Year	2003	2004	2005	2006
Profit (millions)	10,526	11,330	12,715	12,599

Does it produce a reasonable prediction for 2016? Why or why not? (5 points)

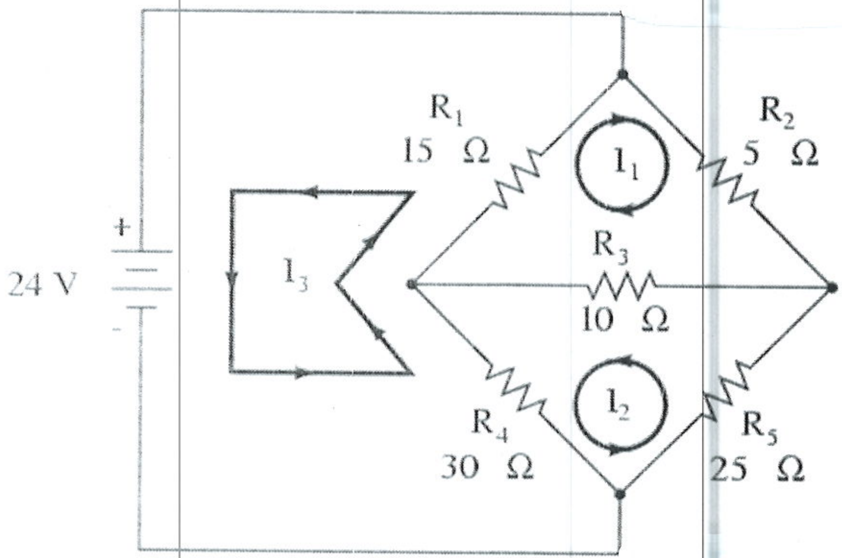
$3^3a + 3^2b + 3c + d = 10,526$
$4^3a + 4^2b + 4c + d = 11,330$
$5^3a + 5^2b + 5c + d = 12,715$
$6^3a + 6^2b + 6c + d = 12,599$

model is dependent w/ these values use years from 2000 instead.

$-347x^3 + 4454.5x^2 - 17538.5x + 32,420 = y$
$y(16) = -529,156$

No. not realistic

3. Set up and solve the loop circuit diagram below. Round your values for the currents to three significant digits. (5 points)



$$30I_1 - 10I_2 - 15I_3 = 0$$
$$-10I_1 + 65I_2 - 30I_3 = 0$$
$$-15I_1 - 30I_2 + 45I_3 = 24$$

$$\vec{I} = \begin{bmatrix} .9379 \\ .7724 \\ 1.3609 \end{bmatrix}$$

4. For $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}$ and $f(x) = x^2 - 5x + 2$, find $f(A)$. (4 points)

$$\begin{bmatrix} 6 & 1 & -3 \\ 0 & 3 & 5 \\ -4 & 2 & 12 \end{bmatrix} - \begin{bmatrix} 10 & 5 & -5 \\ 5 & 0 & 10 \\ -5 & 5 & 15 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -4 & 2 \\ -5 & 5 & -5 \\ 1 & -3 & -1 \end{bmatrix}$$

5. Solve the system $\begin{cases} x_1 + 2x_2 - x_3 + 3x_4 - x_5 = -3 \\ x_1 - 3x_2 + x_3 + 2x_4 - x_5 = -3 \\ 2x_1 + x_2 + x_3 - 3x_4 + x_5 = 6 \\ x_1 - x_2 + 2x_3 + x_4 - x_5 = 2 \\ 2x_1 + x_2 - x_3 + 2x_4 + x_5 = -3 \end{cases}$ by inverse methods. You may find the inverse in your calculator, but multiply $A^{-1}\vec{b}$ by hand. (6 points)

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 & -1 \\ 1 & -3 & 1 & 2 & -1 \\ 2 & 1 & 1 & -3 & 1 \\ 1 & -1 & 2 & 1 & -1 \\ 2 & 1 & -1 & 2 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 22/93 & 11/31 & 34/93 & -8/31 & -1/31 \\ 3/31 & -11/31 & -1/31 & 8/31 & 1/31 \\ -10/31 & -15/31 & -7/31 & 25/31 & 7/31 \\ -4/31 & -6/31 & -9/31 & 10/31 & 9/31 \\ -59/93 & -14/31 & -32/93 & 13/31 & 21/31 \end{bmatrix}$$

$$A^{-1}\begin{bmatrix} -3 \\ -3 \\ 6 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}$$

6. a. Encode the message THE EAGLE HAS LANDED with the matrix $A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & -3 & -1 \\ 3 & 2 & 1 \end{bmatrix}$ using 0 as a space and the corresponding position (number) in the alphabet for letters. (5 points)

message

$$\begin{bmatrix} 20 & 8 & 5 \\ 0 & 5 & 1 \\ 7 & 12 & 5 \\ 0 & 8 & 1 \\ 19 & 0 & 12 \\ 1 & 14 & 4 \\ 5 & 4 & 0 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ -3 & -3 & -1 \\ 3 & 2 & 1 \end{bmatrix} =$$

encoded message

71 26 17 -12 -13 -4 7 -12 0
-21 -22 -7 112 62 31 -26 -32 -9
8 -2 1

b. Devise your own encoding scheme to allow for both letters and positive integers. Describe your scheme here. Then use A to encode the message REPORT TO ROOM 314 AT 212 HUDSON STREET in your scheme. (5 points)

answers will vary

Could use negative #'s for letters and positive #'s for integers (positive) or vice versa

7. Find conditions on $A = \begin{bmatrix} w & x \\ y & z \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ so that $AB = BA$. (5 points)

$AB = \begin{bmatrix} w-x & w+x \\ y-z & y+z \end{bmatrix}$ $\quad BA = \begin{bmatrix} w+y & x+z \\ -w+y & -x+z \end{bmatrix}$

$\cancel{w}-x = \cancel{w}+y \Rightarrow -x = y$

$\cancel{y}-z = -w+\cancel{y} \Rightarrow w = z$

$w+\cancel{x} = \cancel{x}+z \Rightarrow w = z$

$y+\cancel{z} = -x+\cancel{z} \Rightarrow y = -x$

$AB = BA$ for $A = \begin{bmatrix} w & x \\ -x & w \end{bmatrix}$

8. Prove that if the product AB is a square matrix, then BA is defined. (6 points)

if AB is defined then A is $(m \times n)$ and B is $(n \times p)$.
So that AB is $(m \times p)$.

if AB is square, then $m = p$.

So $\Rightarrow$ A is $(m \times n)$ and B is $(n \times m)$

therefore BA is an $(n \times m)$ matrix times $(m \times n)$ and this product is defined since $m = m$.

(The resulting matrix is also square)

9. A nilpotent matrix is a matrix where $A \neq 0$ (the zero matrix), but some power of A does equal the zero matrix. The smallest value of k where $A^k = 0$ is called the index. Determine if $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ is nilpotent, and if so, what is its index? (5 points)

$$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A is nilpotent
it's index is $\boxed{3}$