MAT 202, Final Exam, Part I, Summer 2016 Name KEY

Instructions: Show all work. You may **not** use a calculator on this portion of the exam. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals). Reduce as much as possible. Be sure to complete all parts of each question. Provide explanations where requested. When you are finished with this portion of exam, get Part II.

1. Determine if each statement is True or False. (2 points each)

a. T F An eigenvector may be the zero vector. $\vec{0}$ works for every matrix

b. T F If $\vec{u} \cdot \vec{v} < 0$, then the angle θ between $\vec{u}$ and $\vec{v}$ is acute. obtuse

c. T F If the determinant of an $n \times n$ matrix A is zero, then $A\vec{x} = \vec{0}$ has only the trivial solution. this is true when the det $\neq 0$

d. T F Interchanging two columns of a matrix changes the sign of the determinant.

e. T F If a set of nonzero vectors S in an inner product space V is orthogonal, then S is linearly independent.

f. T F The orthogonal complement of R^n is the empty set. $\{\vec{0}\} = \mathbb{R}^{n\perp}$

g. T F The eigenvectors corresponding to distinct eigenvalues are orthogonal for symmetric matrices.

h. T F If A and B are similar, they must have the same characteristic polynomial.

i. T F The vector spaces R^3 and P_3 are isomorphic to each other. $\mathbb{R}^3$ is isomorphic to P_2

j. T F The fact that A has n distinct eigenvalues does not guarantee A is diagonalizable. it does guarantee its diagonalizable

k. T F All linear transformations T have a unique inverse T^{-1}. only invertible / nonsingular ones

l. T F In general, the compositions $T_2 \circ T_1$ and $T_1 \circ T_2$ have the same standard matrix A. generally, they do not

m. T F If A is an $n \times n$ matrix with an eigenvalue λ, the set of all eigenvectors of λ is a subspace of R^n.

n. T F The matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is onto.

o. T F To find the eigenvalues of an $n \times n$ matrix A, you can solve the characteristic equation $\det(\lambda I - A) = 0$.

p. T F The vector $-\vec{v}$ is called the additive inverse of $\vec{v}$.

q. T F Elementary row operations preserve the row space of the matrix A.

r. T F If A can be row-reduced to the identity, then A is invertible.

s. T F All $n \times n$ matrices are invertible.

t. T F The matrix $A = \begin{bmatrix} 5 & 2 & -1 \\ 2 & 1 & 8 \\ -1 & 8 & 0 \end{bmatrix}$ is orthogonally diagonalizable.

2. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$. (8 points)

$(2-\lambda)(-6-\lambda)-9=0$

$\lambda^2+4\lambda-12-9=0$

$\lambda^2+4\lambda-21=0$

$(\lambda+7)(\lambda-3)=0$

$\lambda=-7, 3$

$\begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix}$ $3x_1+x_2=0$ $\Rightarrow x_1=-\frac{1}{3}x_2$ $x_2=x_2$ $\vec{v_1}=\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$\begin{bmatrix} -1 & 3 \\ 3 & -9 \end{bmatrix}$ $-x_1+3x_2=0$ $x_1=3x_2$ $x_2=x_2$ $\vec{v_2}=\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

3. For the matrix $A = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$ with eigenvalues $\lambda_1 = -1, \lambda_2 = -2$ with corresponding eigenvectors $\vec{v}_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Find the similarity transformation that diagonalizes A and find D. (6 points)

$P=\begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$ $D=\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$

4. Solve the linear system of ODEs given by $\vec{x}' = \begin{bmatrix} 7 & -1 \\ 3 & 3 \end{bmatrix} \vec{x}$. Write the solution in standard form, and plot several sample trajectories. (8 points)

$(7-\lambda)(3-\lambda)+3=0$

$\lambda^2-10\lambda+21+3=0$

$\lambda^2-10\lambda+24=0$

$(\lambda-4)(\lambda-6)=0$

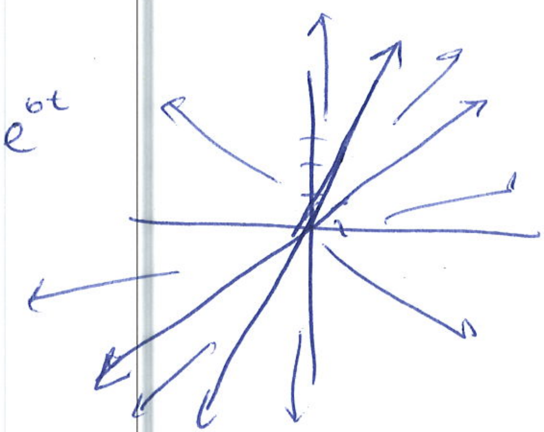
$\lambda=4,6$

$\begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix}$ $3x_1-x_2=0$, $x_1=\frac{1}{3}x_2$, $x_2=x_2$ $\vec{v}_1=\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix}$ $x_1-x_2=0$, $x_1=x_2$, $x_2=x_2$ $\vec{v}_2=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\vec{x}=c_1\begin{bmatrix} 1 \\ 3 \end{bmatrix}e^{4t}+c_2\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{6t}$

Origin repels



5. Find the equilibrium vector for the Markov chain given by the transition matrix $P = \begin{bmatrix} .99 & .08 \\ .01 & .92 \end{bmatrix}$. (7 points)

$P-I=\begin{bmatrix} -.01 & .08 \\ .01 & -.08 \end{bmatrix}$ $.01x_1-.08x_2=0$, $.01x_1=.08x_2$, $x_1=8x_2$, $x_2=x_2$ $\vec{v}=\begin{bmatrix} 8 \\ 1 \end{bmatrix} \Rightarrow q=\begin{bmatrix} 8/9 \\ 1/9 \end{bmatrix}$

6. Given the vector $\vec{u} = \begin{bmatrix} 2 \\ 1 \\ 5 \\ -5 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -1 \\ 2 \\ -1 \\ 1 \end{bmatrix}$, find the following: (3 points each)

a. $||\vec{u}||$

$\sqrt{4+1+25+25} = \sqrt{55}$

b. A unit vector in the direction of $\vec{u}$

$\hat{u} = \begin{bmatrix} 2/\sqrt{55} \\ 1/\sqrt{55} \\ 5/\sqrt{55} \\ -5/\sqrt{55} \end{bmatrix}$

c. $\vec{u} \cdot \vec{v}$

$-2+2-5-5 = -10$

d. Are $\vec{u}$ and $\vec{v}$ orthogonal? If not, is the angle between the vectors acute or obtuse?

no.

the angle is obtuse.

7. For $W = span\left\{\begin{bmatrix}1\\2\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\\-2\end{bmatrix}\right\}$, find the projection of $\vec{y} = \begin{bmatrix}4\\1\\1\\-2\end{bmatrix}$ onto W, and its orthogonal complement in $W^{\perp}$. (8 points)

$$\frac{4+2+0-2}{(\sqrt{6})^2}\begin{bmatrix}1\\2\\0\\1\end{bmatrix} = \frac{4}{6}\begin{bmatrix}1\\2\\0\\1\end{bmatrix} = \begin{bmatrix}2/3\\4/3\\0\\2/3\end{bmatrix}$$

$$\frac{0+1+0+4}{(\sqrt{1+4})^2}\begin{bmatrix}0\\1\\0\\-2\end{bmatrix} = \frac{5}{5}\begin{bmatrix}0\\1\\0\\-2\end{bmatrix} = \begin{bmatrix}0\\1\\0\\-2\end{bmatrix}$$

$$\vec{y}_{\parallel} = \begin{bmatrix}2/3\\4/3\\0\\2/3\end{bmatrix} + \begin{bmatrix}0\\1\\0\\-2\end{bmatrix} = \begin{bmatrix}2/3\\7/3\\0\\-4/3\end{bmatrix} \text{ in } W$$

$$\vec{y}_{\perp} = \begin{bmatrix}4\\1\\1\\-2\end{bmatrix} - \begin{bmatrix}2/3\\7/3\\0\\-4/3\end{bmatrix} = \begin{bmatrix}10/3\\-4/3\\1\\-2/3\end{bmatrix} \text{ in } W^{\perp}$$

8. Use any method to find the determinant of $\begin{bmatrix}1 & 2 & -1 & 4\\0 & 1 & 2 & -2\\0 & 3 & 2 & -1\\1 & 2 & 0 & 0\end{bmatrix}$. (8 points)

$-R_1 + R_4 \to R_4$ $\begin{bmatrix}1 & 2 & -1 & 4\\0 & 1 & 2 & -2\\0 & 3 & 2 & -1\\0 & 0 & 1 & -4\end{bmatrix}$

$$1\begin{vmatrix}1 & 2 & 2\\3 & 2 & -1\\0 & 1 & -4\end{vmatrix} = 1\begin{vmatrix}2 & -1\\1 & -4\end{vmatrix} - 3\begin{vmatrix}2 & -2\\1 & -4\end{vmatrix} + \cancel{0\begin{vmatrix}2 & 2\\2 & -1\end{vmatrix}}$$

$$(-8+1) - 3(-8+2) = -7 - 3(-6) =$$

$$-7 + 18 = \boxed{11}$$

9. Use properties of determinants and $A = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ -2 & 5 \end{bmatrix}$ to find: (3 points each)

$\det A = -1 + 6 = 5$

$\det B = -5 + 4 = -1$

a. $\det(AB)$ $(5)(-1) = -5$

b. $\det(A^2)$ $(5)^2 = 25$

c. $\det(3A)$ $3^2(5) = 9(5) = 45$

d. $\det(A^{-1}B^T)$ $\frac{1}{5}(-1) = -\frac{1}{5}$

10. Determine by inspection, if each set of vectors is linearly independent. Explain your reasoning. (3 points each)

a. $\left\{\begin{bmatrix} -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix}\right\}$ independent, since there are only two vectors and they are not multiples of each other

b. $\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}\right\}$ independent. There is a pivot in every column

c. $\{x^2 - 1, 2x + 5\}$ independent; 2 vectors, not multiples

11. For the matrices $A = \begin{bmatrix} -4 & 3 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}$, $D = \begin{bmatrix} 3 & 2 & 1 \\ -3 & 0 & 4 \\ 4 & -2 & -4 \end{bmatrix}$.

Calculate the following matrices. If the operation is not defined, explain why not. (4 points each)

a. CC^T

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 2 & -3 & 1 \\ 1 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 4+1 & -6+4 & 2+6 \\ -6+4 & 9+16 & -3+24 \\ 2+6 & -3+24 & 1+36 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 8 \\ -2 & 25 & 21 \\ 8 & 21 & 37 \end{bmatrix}$$

b. BC $(2\times2)(3\times2)$

not defined inner dimensions don't match

MAT 202, Final Exam, Part II, Summer 2016 Name ____________________

Instructions: Show all work. You **may** use a calculator on this portion of the exam. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question. Provide explanations where requested.

1. Find e^A for $A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$. (8 points)

$(4-\lambda)(1-\lambda)+2 = \lambda^2 - 5\lambda + 4 + 2 = 0$

$\lambda^2 - 5\lambda + 6 = 0$

$(\lambda-3)(\lambda-2) = 0$

$\lambda = 3, 2$

$\begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$ $x_1 - 2x_2 = 0$, $x_1 = 2x_2$, $x_2 = x_2$ $\Rightarrow \vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$ $x_1 = x_2$, $x_2 = x_2$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

$e^D = \begin{bmatrix} e^3 & 0 \\ 0 & e^2 \end{bmatrix}$

$e^A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^3 & 0 \\ 0 & e^2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^3 & -e^3 \\ -e^2 & 2e^2 \end{bmatrix} = \begin{bmatrix} 2e^3 - e^2 & -2e^3 + 2e^2 \\ e^3 - e^2 & -e^3 + 2e^2 \end{bmatrix}$

2. Solve the discrete dynamical system $A = \begin{bmatrix} 1.24 & -.97 \\ 1 & 0 \end{bmatrix}, \vec{x_0} = \begin{bmatrix} -2 \\ 12 \end{bmatrix}$. Find and plot 10 points along the trajectory starting at the indicated point. What is the long-term behavior of the system starting at any initial condition? Write your solution in terms of the eigenvectors and eigenvalues. (8 points)

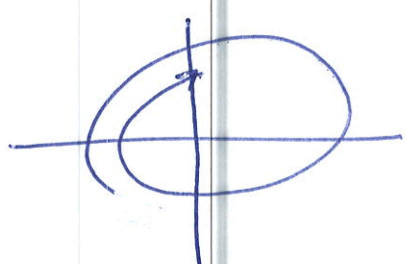
$(1.24-\lambda)(-\lambda) + .97 =$

$\lambda^2 - 1.24\lambda + .97 = 0$

$\dfrac{1.24 \pm \sqrt{1.24^2 - 4(.97)}}{2} =$

$\dfrac{1.24 \pm \sqrt{-2.3424}}{2} =$

$.62 \pm .765245i$

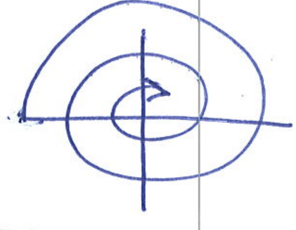


origin spirals inward

$x_0 = \begin{bmatrix} -2 \\ 12 \end{bmatrix}, x_1 = \begin{bmatrix} -14.12 \\ -2 \end{bmatrix}, x_2 = \begin{bmatrix} -15.57 \\ -14.12 \end{bmatrix}$ $x_3 = \begin{bmatrix} -5.61 \\ -15.57 \end{bmatrix},$

$x_4 = \begin{bmatrix} 8.15 \\ -5.61 \end{bmatrix}, x_5 = \begin{bmatrix} 15.54 \\ 8.15 \end{bmatrix}, x_6 = \begin{bmatrix} 11.37 \\ 15.54 \end{bmatrix}, x_7 = \begin{bmatrix} -.98 \\ 11.37 \end{bmatrix},$

$x_8 = \begin{bmatrix} -12.24 \\ -.98 \end{bmatrix}, x_9 = \begin{bmatrix} -14.23 \\ -12.24 \end{bmatrix}, x_{10} = \begin{bmatrix} -5.77 \\ -14.23 \end{bmatrix}$



$\sqrt{.62^2 + .765^2} = \sqrt{.9696} < 1$

3. The table below shows the revenue (in billions of dollars) for the General Dynamics Corporation from 2005 to 2010. Graph the points. Select a polynomial model for the data and find a regression equation of best fit. Use that equation to predict revenue for 2016. Does your prediction make sense? (8 points)

Year	2005	2006	2007	2008	2009	2010
Revenue (y)	21.2	24.1	27.2	29.3	32.0	32.5

use years since 2000 for x

(using quadratic model)
linear might also be okay

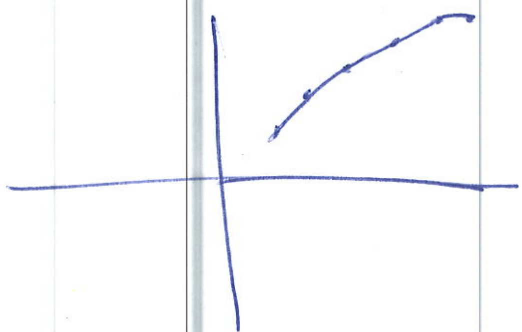
$$A = \begin{bmatrix} 1 & 5 & 25 \\ 1 & 6 & 36 \\ 1 & 7 & 49 \\ 1 & 8 & 64 \\ 1 & 9 & 81 \\ 1 & 10 & 100 \end{bmatrix} \quad \vec{y} = \begin{bmatrix} 21.2 \\ 24.1 \\ 27.2 \\ 29.3 \\ 32.0 \\ 32.5 \end{bmatrix}$$

$y = \beta_0 + \beta_1 x + \beta_2 x^2$

$y = -2.87 + 5.99x - .243x^2$

$y(16) = 30.762$

yes, it makes sense



linear:

$$\begin{bmatrix} 10.08 \\ 2.35 \end{bmatrix}$$

$y = 10.08 + 2.35x$

4. Prove or disprove that if λ is an eigenvalue of A, it is also an eigenvalue of A^2. (5 points)

Consider $A\vec{x} = \lambda\vec{x}$ for some λ

$A(A\vec{x}) = A(\lambda\vec{x})$

$A^2\vec{x} = \lambda A\vec{x} = A^2\vec{x} = \lambda(\lambda\vec{x}) = \lambda^2\vec{x}$

it is false that if λ is an eigenvalue of A, it is also an eigenvalue of A^2. λ^2 is an eigenvalue of A^2

5. Given the linear transformation defined by $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 3 & 1 & 2 & -1 \\ -4 & -3 & -1 & -3 \\ -1 & -2 & 1 & 1 \end{bmatrix}$, determine if the transformation is any of the following. Explain your reasoning in each case. (3 points each)

a. One-to-one

rref $\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

no; it does not have a pivot in every column

b. Onto

no; it does not have a pivot in every row

c. An isomorphism from R^4 to R^4.

no; it must be both one-to-one and onto to be an isomorphism

6. Consider a generic 8×7 matrix. Is it possible for the linear transformation defined by the matrix to be:

a. One-to-one? (3 points)

yes. Since there are 7 columns, it can have 7 pivots

b. Onto? (3 points)

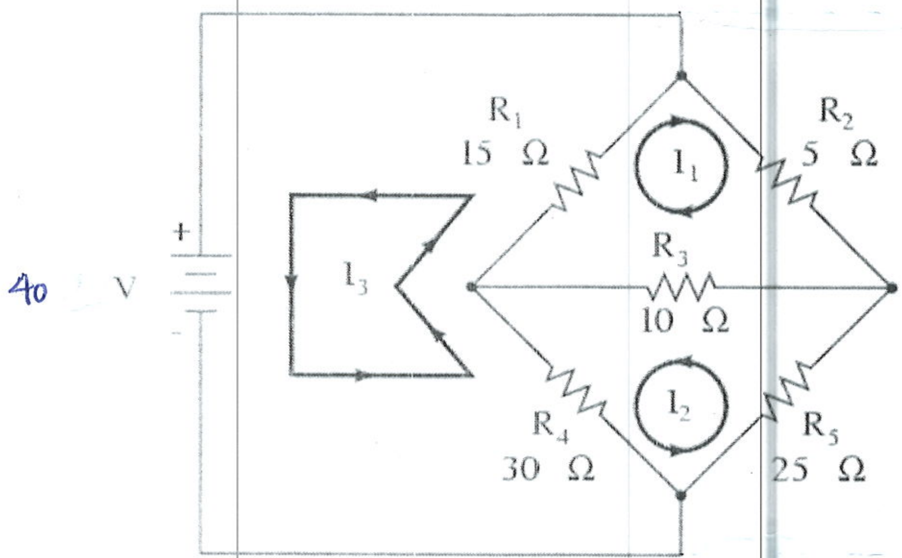
no; it can have a max of 7 pivots so one row will not have one

c. If the matrix has 5 pivots, what is the dimension of the kernel and the range? (4 points)

$\dim(\ker(A)) = 2$

$\dim(\text{range}(A)) = 5$

7. Set up and solve the loop circuit diagram below. Round your values for the currents to three significant digits. (8 points)



$$30I_1 - 10I_2 - 15I_3 = 0$$
$$-10I_1 + 65I_2 - 30I_3 = 0$$
$$-15I_1 - 30I_2 + 45I_3 = 40$$

$$\vec{I} = \begin{bmatrix} 1.56 \\ 1.29 \\ 2.27 \end{bmatrix}$$

8. Find conditions on $A = \begin{bmatrix} w & x \\ y & z \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$ so that $AB = BA$. (8 points)

$$\begin{bmatrix} w-2x & 3w+x \\ y-2z & 3y+z \end{bmatrix} = \begin{bmatrix} w+3y & x+3z \\ -2w+y & -2x+z \end{bmatrix}$$

$$A = \begin{bmatrix} w & -\frac{3}{2}y \\ y & w \end{bmatrix}$$

$\cancel{w}-2x = \cancel{w}+3y$

$\cancel{y}-2z = -2w+\cancel{y}$

$z = w$

$3w+\cancel{x} = \cancel{x}+3z$

$w = z$

$3y+\cancel{z} = -2x+\cancel{z}$

$y = -\frac{2}{3}x$

9. Find the volume of the parallelepiped whose corner is defined by the vectors $\begin{bmatrix}1\\-2\\3\end{bmatrix}, \begin{bmatrix}4\\0\\1\end{bmatrix}, \begin{bmatrix}2\\5\\-3\end{bmatrix}$. (4 points)

$$\begin{vmatrix} 1 & 4 & 2 \\ -2 & 0 & 5 \\ 3 & 1 & -3 \end{vmatrix} = 27$$

10. Determine whether $S = \{t^3 - 1, 2t^2, t+3, 5 + 2t + 2t^2 + t^3\}$ is a basis for P_3. (4 points)

$$\begin{bmatrix} -1 & 0 & 3 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ no, it is not a basis for P_3

11. Consider the matrix $A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$. Find a basis for:

a. The nullspace of A (4 points)

$$\text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Nul } A = \text{span}\left\{ \begin{bmatrix}-1\\2\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\-3\\0\\5\\1\end{bmatrix} \right\}$$

$x_1 = -x_3 - x_5$
$x_2 = 2x_3 - 3x_5$
$x_3 = x_3$
$x_4 = 5x_5$
$x_5 = x_5$

b. The column space of A (3 points)

$$\text{Col } A = \text{span}\left\{\begin{bmatrix}-2\\1\\3\\1\end{bmatrix}, \begin{bmatrix}-5\\3\\11\\7\end{bmatrix}, \begin{bmatrix}0\\1\\7\\5\end{bmatrix}\right\}$$

c. The rank of A (2 points)

$$\text{rank } A = 3$$

12. Find the change of basis matrix to transition from C to B for $B = \left\{\begin{bmatrix}1\\3\\2\\-1\end{bmatrix}, \begin{bmatrix}-2\\-5\\-5\\4\end{bmatrix}, \begin{bmatrix}-1\\-2\\-2\\4\end{bmatrix}, \begin{bmatrix}-2\\-3\\-5\\11\end{bmatrix}\right\}$, $C = \left\{\begin{bmatrix}1\\2\\4\\-1\end{bmatrix}, \begin{bmatrix}-2\\-3\\4\\2\end{bmatrix}, \begin{bmatrix}0\\1\\2\\-2\end{bmatrix}, \begin{bmatrix}1\\-1\\0\\1\end{bmatrix}\right\}$. Use it to convert $[\vec{x}]_C = \begin{bmatrix}0\\-2\\4\\5\end{bmatrix}$ to $[\vec{x}]_B$. (6 points)

$$P_C[\vec{x}]_C = P_B[\vec{x}]_B$$

$$\underbrace{P_B^{-1}P_C}_{P_{B\leftarrow C}}[\vec{x}]_C = [\vec{x}]_B$$

$$\begin{bmatrix}-167\\-68\\-212\\86\end{bmatrix} = \begin{bmatrix}-4 & 27 & 13 & -33\\-3 & 9 & 5 & -14\\-1 & 45 & 17 & -38\\1 & -17 & -7 & 16\end{bmatrix}\begin{bmatrix}0\\-2\\4\\5\end{bmatrix}$$

13. Solve the system $\begin{cases}4x_1 - x_2 + x_3 = -5\\2x_1 + 2x_2 + 3x_3 = 10\\5x_1 - 2x_2 + 6x_3 = 1\end{cases}$ by any method. (7 points)

$$\left[\begin{array}{ccc|c}4 & -1 & 1 & -5\\2 & 2 & 3 & 10\\5 & -2 & 6 & 1\end{array}\right] \Rightarrow \text{rref} \Rightarrow \left[\begin{array}{ccc|c}1 & 0 & 0 & -1\\0 & 1 & 0 & 3\\0 & 0 & 1 & 2\end{array}\right]$$

$$\vec{x} = \begin{bmatrix}-1\\3\\2\end{bmatrix}$$