

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. For each set of vectors, determine if the set is linearly independent. If it is not, find a spanning set for the subspace.

a. $\begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$

yes. not multiples; therefore a set of 2 vectors are independent.

b. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -6 \end{bmatrix}$

no (dependent) there are more vectors than are needed to span the space.

c. $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$

rref $\Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ dependent; no pivot in last column

2. Write the standard basis for P_4 . State the dimension of the space.

$$x^4, x^3, x^2, x, 1$$

$$\dim P_4 = 5$$

3. Determine whether $\{4 - t, t^3, 6t^2, t^3 + 3t, 4t - 1\}$ is a basis for P_3 . If it is not, explain why not.

$$\begin{bmatrix} 4 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 3 & 4 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

no, it is not a basis

$\dim(P_3) = 4$ & there are 5 vectors

So these are not linearly independent

rref $\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -1/4 \\ 0 & 1 & 0 & 0 & -3/4 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3/4 \end{bmatrix}$

no pivot in last column
So not independent