MAT 202, Quiz #15, Summer 2016 Name KEY

**Instructions**: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Determine whether $T\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) = \begin{bmatrix} 4v_2 - v_1 \\ 4v_1 + 5v_2 \\ v_2 - v_3 \end{bmatrix}$ is a linear transformation. Prove it if it is; if it is not, find an example of where the definition fails and state which property is violated.

this transformation is linear

a) if $v_1 = v_2 = v_3 = 0$ $T(\vec{0}) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ✓

b) $T\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) + T\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = \begin{bmatrix} 4v_2 - v_1 \\ 4v_1 + 5v_2 \\ v_2 - v_3 \end{bmatrix} + \begin{bmatrix} 4u_2 - u_1 \\ 4u_1 + 5u_2 \\ u_2 - u_3 \end{bmatrix} = \begin{bmatrix} 4(v_2+u_2) - (v_1+u_1) \\ 4(v_1+u_1) + 5(v_2+u_2) \\ (v_2+u_2) - (v_3+u_3) \end{bmatrix}$

$T\left(\begin{bmatrix} v_1+u_1 \\ v_2+u_2 \\ v_3+u_3 \end{bmatrix}\right) = \begin{bmatrix} 4(v_2+u_2) - (v_1+u_1) \\ 4(v_1+u_1) + 5(v_2+u_2) \\ (v_2+u_2) - (v_3+u_3) \end{bmatrix}$ ✓

c) $kT\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) = k\begin{bmatrix} 4v_2 - v_1 \\ 4v_1 + 5v_2 \\ v_2 - v_3 \end{bmatrix} = \begin{bmatrix} 4kv_2 - kv_1 \\ 4kv_1 + 5kv_2 \\ kv_2 - kv_3 \end{bmatrix} = T\left(\begin{bmatrix} kv_1 \\ kv_2 \\ kv_3 \end{bmatrix}\right)$ ✓

2. For the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 3 & 1 & 2 & -1 \\ -4 & -3 & -1 & 3 \\ -1 & -2 & 1 & 1 \end{bmatrix}$ of the linear transformation $T(\vec{v}) = A\vec{v}$, find

rref $\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

a. *Col A*

$\left\{ \begin{bmatrix} 1 \\ 3 \\ -4 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 3 \\ 1 \end{bmatrix} \right\}$

b. *Nul A*

$\left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

$x_1 = -x_3$
$x_2 = x_3$
$x_3 = x_3$
$x_4 = 0$
$\Rightarrow \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

c. *Rank A*

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d. Is the transformation an isomorphism from $R^4 \to R^4$?

no it is not since it is not one-to-one or onto