MAT 202, Quiz #23, Summer 2016 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. A Markov chain is defined by the transition matrix $P = \begin{bmatrix} .5 & .04 \\ .5 & .96 \end{bmatrix}$. Find the steady-state vector of the system.

$P - I = \begin{bmatrix} -.5 & .04 \\ .5 & -.04 \end{bmatrix} \Rightarrow$ $.5x_1 = .04x_2$

$x_1 = .08x_2$

$x_2 = x_2$

$\vec{x} = \begin{bmatrix} .08 \\ 1 \end{bmatrix} \Rightarrow \vec{q} = \begin{bmatrix} 2/27 \\ 25/27 \end{bmatrix}$

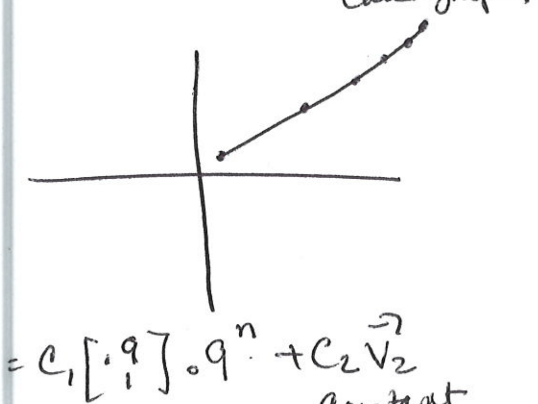
$\div 1.08$

2. A discrete dynamical system is given by $A = \begin{bmatrix} 1.8 & -.81 \\ 1 & 0 \end{bmatrix}$, $\vec{x_0} = \begin{bmatrix} 15 \\ 3 \end{bmatrix}$. Plot 5 points of the trajectory of the system. Then solve the system and write the solution in the form $\vec{x}_n = c_1\vec{v}_1\lambda_1^n + c_2\vec{v}_2\lambda_2^n$. Use that information to plot some additional sample trajectories and describe the character of the origin.

$\vec{x_0} = \begin{bmatrix} 15 \\ 3 \end{bmatrix}$ $\vec{x_1} = \begin{bmatrix} 24.57 \\ 15 \end{bmatrix}$. $\vec{x_2} = \begin{bmatrix} 32.076 \\ 24.57 \end{bmatrix}$ $\vec{x_3} = \begin{bmatrix} 37.8351 \\ 32.076 \end{bmatrix}$

$\vec{x_4} = \begin{bmatrix} 42.12162 \\ 37.8351 \end{bmatrix}$, $\vec{x_5} = \begin{bmatrix} 45.17\ldots \\ 42.12\ldots \end{bmatrix}$

Calc graph.



$\begin{bmatrix} 1.8-\lambda & -.81 \\ 1 & 0-\lambda \end{bmatrix} \Rightarrow (1.8-\lambda)(-\lambda) + .81 = 0$

$\lambda^2 - 1.8\lambda + .81 = 0$

$(\lambda - .9)^2 = 0 \quad \lambda = .9$

$\begin{bmatrix} .9 & -.81 \\ 1 & -.9 \end{bmatrix}$ $x_1 - .9x_2 = 0$

$\Rightarrow x_1 = .9x_2$ $\vec{v_1} = \begin{bmatrix} .9 \\ 1 \end{bmatrix}$

$x_2 = x_2$

$\vec{x_n} = c_1 \begin{bmatrix} .9 \\ 1 \end{bmatrix} .9^n + c_2 \underbrace{\vec{v_2}}_{\text{Constant}}$

only one eigenvector

v_2 will depend on initial state vector