212 Homework #10 key

1a. $\langle f, g\rangle = \int_{-\pi}^{\pi} f(x)g(x)\,dx$

$\langle \sin(nx), \sin(mx)\rangle = \int_{-\pi}^{\pi} \sin(nx)\sin(mx)\,dx$

$\sin a \sin b = \frac{1}{2}(\cos(a-b) - \cos(a+b))$

$\frac{1}{2}\int_{-\pi}^{\pi} \cos[(n-m)x] - \cos[(m+n)x]\,dx$ if $n \neq m$

$= \frac{1}{2}\left[\frac{\sin[n-m]x}{n-m} - \frac{\sin[m+n]x}{m+n}\right|_{-\pi}^{\pi}$

$n, m \in \mathbb{Z}$
$n-m = k \in \mathbb{Z}$
$n+m = p \in \mathbb{Z}$

$\frac{1}{2}\left[\cancel{\frac{\sin k\pi}{k}}^{0} - \cancel{\frac{\sin p\pi}{p}}^{0}\right] = 0$ orthogonal

if $n = m$

$\frac{1}{2}\int_{-\pi}^{\pi} \cos(0) - \cos(2n)\,dx = \frac{1}{2}\left[x - \frac{\sin 2n}{n}\right|_{-\pi}^{\pi} = \pi$ not orthogonal w/ itself

b. $\cos(nx), \cos(mx)$ $\cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$

$\int_{-\pi}^{\pi} \cos nx \cdot \cos mx\,dx = \frac{1}{2}\int_{-\pi}^{\pi} \cos(n+m)x + \cos(m-n)x\,dx$

if $m \neq n$

$\frac{1}{2}\left[\frac{\sin(n+m)}{n+m} + \frac{\sin(n-m)x}{n-m}\right]_{-\pi}^{\pi}$

$n \in \mathbb{Z}, m \in \mathbb{Z}$
$n \pm m \in \mathbb{Z}$, $n+m = k \in \mathbb{Z}$
$n-m = p \in \mathbb{Z}$

$\frac{1}{2}\left[\cancel{\frac{\sin k\pi}{k}}^{0} + \cancel{\frac{\sin p\pi}{p}}^{0} \dots\right] = 0$ orthogonal if $m \neq n$

if $m = n$

$\frac{1}{2}\int_{-\pi}^{\pi} \cos 2n + \underbrace{\cos 0}_{=1}\,dx = \frac{1}{2}\left[\frac{\sin 2nx}{2n} + x\right]_{-\pi}^{\pi} = \pi$ not orthogonal to itself

c. $\langle \sin nx, \cos mx\rangle$ $\frac{1}{2}(\sin(a+b) + \sin(a-b))$

$\int_{-\pi}^{\pi} \sin(nx)\cos(mx)\,dx = \frac{1}{2}\int_{-\pi}^{\pi} \sin(n+m)x + \sin(n-m)x\,dx =$ $n \neq m$

$n, m \in \mathbb{Z} \Rightarrow n+m, n-m \in \mathbb{Z}$

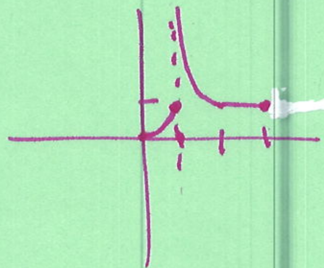
$\frac{1}{2}\left[\frac{-\cos(n+m)x}{n+m} - \frac{\cos(n-m)x}{n-m}\right]_{-\pi}^{\pi} = \frac{-1}{2}\left[\frac{\cos k\pi}{k} + \frac{\cos p\pi}{p} - \frac{\cos k\pi}{k} - \frac{\cos p\pi}{p}\right] = 0$

1c. cont'd

Orthogonal if $n \neq m$

if $m=n$ $\frac{1}{2}\int_{-\pi}^{\pi} \sin 2nx + 0 \, dx = \frac{1}{2}\left[\frac{\cos 2nx}{2n}\right]_{-\pi}^{\pi} = 0$ orthogonal

2. $f(t) = \begin{cases} t^2 & 0 \le t \le 1 \\ (t-1)^{-1} & 1 < t \le 2 \\ 1 & 2 < t \le 3 \end{cases}$



piecewise continuous

3a. $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\, dt$

$f(t) = t$ $\quad \int_0^{\infty} t e^{-st} dt$

$\pm$	u	dv
$+$	t	e^{-st}
$-$	1	$-\frac{1}{s}e^{-st}$
$+$	0	$+\frac{1}{s^2}e^{-st}$

note: $\frac{t^n}{e^{-t}} \to 0$ as $t \to \infty$

$= \left[-\frac{1}{s}te^{-st} - \frac{1}{s^2}e^{-st}\right]_0^{\infty} =$

$0 + 0 - 0 + \frac{1}{s^2} = \boxed{\frac{1}{s^2}}$

b. $f(t) = \sinh bt = \frac{e^{bt} - e^{-bt}}{2}$

$\int_0^{\infty} e^{-st} \sinh bt \, dt = \frac{1}{2}\int_0^{\infty} e^{-st}\left[e^{bt} - e^{-bt}\right] dt = \frac{1}{2}\int_0^{\infty} e^{-t(s-b)} - e^{-t(s+b)} \, dt$

$\frac{1}{2}\left[\frac{e^{-t(s-b)}}{-(s-b)} - \frac{e^{-t(s+b)}}{-(s+b)}\right]_0^{\infty} = \frac{1}{2}\left[0 + \frac{1}{s-b} + 0 - \frac{1}{s+b}\right] =$

$\frac{1}{2}\left[\frac{s+b-(s-b)}{(s-b)(s+b)}\right] = \frac{1}{2}\left[\frac{s+b-s+b}{(s-b)(s+b)}\right] = \frac{1}{2}\left[\frac{2b}{s^2-b^2}\right] = \frac{b}{s^2-b^2}$

c. $f(t) = t^2 \sinh bt$

$\int_0^{\infty} t^2 \sinh bt \cdot e^{-st} dt = \frac{1}{2}\int_0^{\infty} t^2\left[e^{-t(s-b)} - e^{-t(s+b)}\right] dt$

$\pm$	u	dv
$+$	t^2	$e^{-t(s-b)} - e^{-t(s+b)}$
$-$	$2t$	$\frac{e^{-t(s-b)}}{-(s-b)} + \frac{e^{-t(s+b)}}{s+b}$
$+$	2	$\frac{e^{-t(s-b)}}{(s-b)^2} - \frac{e^{-t(s+b)}}{(s+b)^2}$
$-$	0	$\frac{e^{-t(s-b)}}{-(s-b)^3} + \frac{e^{-t(s+b)}}{(s+b)^3}$

3c cont'd

$$\frac{1}{2}\left[t^2\left(\frac{e^{-t(s-b)}}{-(s-b)}+\frac{e^{-t(s+b)}}{(s+b)}\right)-2t\left(\frac{e^{-t(s-b)}}{(s-b)^2}-\frac{e^{-t(s+b)}}{(s+b)^2}\right)+2\left(\frac{e^{-t(s-b)}}{-(s-b)^3}+\frac{e^{-t(s+b)}}{(s+b)^3}\right)\right]_0^\infty$$

$$\frac{1}{2}\left[0-0+0-0-0+0+0-0+0+\frac{2}{(s-b)^3}+0-\frac{2}{(s+b)^3}\right]=0$$

$$=\frac{1}{(s-b)^3}-\frac{1}{(s+b)^3}=\frac{(s+b)^3-(s-b)^3}{(s-b)^3(s+b)^3}=\frac{\cancel{s^3}+3s^2b+\cancel{3sb^2}+b^3-(\cancel{s^3}-3s^2b+\cancel{3sb^2}-b^3)}{(s^2-b^2)^3}$$

$$=\frac{6s^2b+2b^3}{(s^2-b^2)^3}=\frac{2b(3s^2+b^2)}{(s^2-b^2)^3}$$

d. $f(t)=\cos at$

note: you can also integrate $\int e^{-st}\cos at\,dt$ by rewriting cosine as $\cos at=\frac{e^{ait}+e^{-ait}}{2}$ and follow pattern used for sinh bt.

$$\int_0^\infty e^{-st}\cos at\,dt$$

$u=\cos at$ $\quad dv=e^{-st}$

$du=-a\sin at$ $\quad v=-\frac{1}{s}e^{-st}$

$$=-\frac{1}{s}\cos(at)e^{-st}\Big|_0^\infty+\int_0^\infty\frac{a}{s}\sin at\cdot e^{-st}\,dt$$

$u=-\frac{a}{s}\sin at$ $\quad dv=e^{-st}$

$du=-\frac{a^2}{s}\cos at$ $\quad v=-\frac{1}{s}e^{-st}$

$$=-\frac{1}{s}e^{-st}\cos at\Big|_0^\infty+\frac{a}{s^2}\sin at\cdot e^{-st}\Big|_0^\infty-\int_0^\infty\frac{a^2}{s^2}e^{-st}\cos at\,dt=\int_0^\infty e^{-st}\cos at\,dt$$

$$+\int\frac{a^2}{s^2}e^{-st}\cos at\,dt\quad+\int_0^\infty\frac{a^2}{s^2}e^{-st}\cos at\,dt$$

$$\Rightarrow -\frac{1}{s}e^{-st}\cos at+\frac{a}{s^2}\sin at\cdot e^{-st}\Big|_0^\infty=\underbrace{\left(1+\frac{a^2}{s^2}\right)}_{\frac{s^2+a^2}{s^2}}\int_0^\infty e^{-st}\cos at\,dt$$

$$\Rightarrow\frac{s^2}{s^2+a^2}\left[-\frac{1}{s}e^{-st}\cos at+\frac{a}{s^2}\sin at\cdot e^{-st}\Big|_0^\infty=\int_0^\infty e^{-st}\cos at\,dt\right.$$

$$=\frac{s^2}{s^2+a^2}\left[0+\frac{1}{s}(1)(1)+\frac{a}{s^2}(0)-\frac{a}{s^2}(0)(1)\right]=\frac{1}{s}\cdot\frac{s^2}{s^2+a^2}=\boxed{\frac{s}{s^2+a^2}}$$

3e. $f(t) = te^{at}$

$$\int_0^\infty e^{-st} \cdot te^{at}\,dt = \int_0^\infty t e^{-t(s-a)}\,dt$$

+/-	u	dv
+	t	$e^{-t(s-a)}$
−	1	$\frac{-1}{s-a}e^{-t(s-a)}$
+	0	$\frac{1}{(s-a)^2}e^{-t(s-a)}$

$$= \frac{-t}{s-a}e^{-t(s-a)} - \frac{1}{(s-a)^2}e^{-t(s-a)}\Big|_0^\infty$$

$$0 + 0 - 0 + \frac{1}{(s-a)^2} = \frac{1}{(s-a)^2}$$

f. $f(t) = \begin{cases} 0 & t < c \\ 1 & t \geq c \end{cases}$ $\quad \int_0^\infty f(t)e^{-st}\,dt = \underbrace{\int_0^c 0 \cdot e^{-st}\,dt}_{=0} +$

$$\int_c^\infty 1 \cdot e^{-st}\,dt = -\frac{1}{s}e^{-st}\Big|_c^\infty = 0 + \frac{1}{s}e^{-sc} = \boxed{\frac{e^{-sc}}{s}}$$

4a. $F(s) = \frac{3}{s^2+4}$ $\qquad \mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2} \quad a = 2$

$$= \frac{3}{2}\left(\frac{2}{s^2+4}\right) = \boxed{\frac{3}{2}\sin 2t}$$

b. $\frac{8s^2-4s+12}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4} = \frac{As^2+4A+Bs^2+Cs}{s(s^2+4)}$

$A + B = 8 \Rightarrow B = 5$

$C = -4$

$4A = 12 \Rightarrow A = 3$

$$\frac{3}{s} + \frac{5s}{s^2+4} - \frac{4}{s^2+4}$$

$$= 3\left(\frac{1}{s}\right) + 5\left(\frac{s}{s^2+4}\right) - 2\left(\frac{s}{s^2+4}\right) =$$

$$\boxed{3 + 5\cos 2t - 2\sin 2t}$$

c. $F(s) = \frac{3s}{s^2-s-6} = \frac{A}{s-3} + \frac{B}{s+2} = \frac{As+2A+Bs-3B}{(s-3)(s+2)}$

$(s-3)(s+2)$

$A + B = 3$

$2A - 3B = 0$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & -3 & 0 \end{array}\right]$$

$A = 9/5$

$B = 6/5$

$$\frac{9}{5}\left(\frac{1}{s-3}\right) + \frac{6}{5}\left(\frac{1}{s+2}\right)$$

$$\boxed{\frac{9}{5}e^{3t} + \frac{6}{5}e^{-2t}}$$

4d. $\dfrac{2s-3}{s^2+2s+10} = \dfrac{2s-3}{(s+1)^2+9} = \dfrac{2(s+1)-5}{(s+1)^2+9} = 2\left(\dfrac{s+1}{(s+1)^2+9}\right) - \dfrac{5}{3}\left(\dfrac{3}{(s+1)^2+9}\right)$

$(s^2+2s+1)+9$

$2(s+1)-5 = 2s+2-5 = 2s-3$

$= \boxed{2e^{-t}\cos 3t - \frac{5}{3}e^{-t}\sin 3t}$

5a. $y''-y'-6y=0 \quad y(0)=1,\ y'(0)=-1$

$s^2Y(s) - s(1) + 1 - (sY(s)-1) - 6(Y(s)) = 0$

$Y(s)(s^2-s-6) - s + 2 = 0 \Rightarrow \dfrac{s-2}{s^2-s-6} = Y(s)$

$\dfrac{s-2}{(s-3)(s+2)} = \dfrac{A}{s-3} + \dfrac{B}{s+2} = \dfrac{As+2A+Bs-3B}{(s-3)(s+2)}$

$A+B=1$
$2A-3B=-2$
$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & -3 & -2 \end{array}\right]$ $A = 1/5$, $B = 4/5$ $\quad Y(s) = \frac{1}{5}\left(\frac{1}{s-3}\right) + \frac{4}{5}\left(\frac{1}{s+2}\right)$

$y(t) = \frac{1}{5}e^{3t} + \frac{4}{5}e^{-2t}$

b. $y^{iv} - y = 0 \quad y(0)=1,\ y'(0)=0,\ y''(0)=1,\ y'''(0)=0$

$s^4Y(s) - s^3(1) - s^2(0) - s(1) - 0 - (Y(s)) = 0$

$Y(s)(s^4-1) = s^3+s \Rightarrow Y(s) = \dfrac{s^3+s}{s^4-1} = \dfrac{s^3+s = s(\cancel{s^2+1})}{(\cancel{s^2+1})(s-1)(s+1)}$

$\dfrac{s}{(s-1)(s+1)} = \dfrac{s}{s^2-1} = \cosh t$

c. $y''+2y'+y = 4e^{-t},\ y(0)=2,\ y'(0)=-1$

$s^2Y(s) - 2s + 1 + 2(sY(s)-2) + Y(s) = 4\cdot\dfrac{1}{s+1}$

$Y(s)(s^2+2s+1) - 2s - 3 = \dfrac{4}{s+1}$

$Y(s) = \dfrac{2s+3}{s^2+2s+1} + \dfrac{4}{(s+1)(s^2+2s+1)} = \dfrac{2s^2+5s+2s+5+4}{(s+1)(s^2+2s+1)}$

$\dfrac{2s^2-3s-1}{(s+1)(s+1)^2} = \dfrac{A}{s+1} + \dfrac{B}{(s+1)^2} + \dfrac{C}{(s+1)^3}$

5c. cont'd

$$\frac{As^2+2As+A+Bs+B+C}{(s+1)^3} \Rightarrow \begin{array}{l} A=2 \\ 2A+B=-3 \Rightarrow 4+B=-3 \Rightarrow B=-7 \\ A+B+C=1 \Rightarrow 2-7+C=1 \\ \quad -5+C=1 \Rightarrow C=6 \end{array}$$

$$\frac{2}{s+1}-\frac{7}{(s+1)^2}+\frac{6}{(s+1)^3}=2\left(\frac{1}{s+1}\right)-7\left(\frac{1}{(s+1)^2}\right)+\frac{6}{2}\left(\frac{2}{(s+1)^3}\right)$$

$$y(t)=2e^{-t}-7te^{-t}+3t^2e^{-t}$$

5d. $y''+4y=\begin{cases}1, & 0\le t\le\pi \\ 0, & \pi\le t<\infty\end{cases}$, $y(0)=1$, $y'(0)=0$

$1-u_\pi(t)$

$$s^2Y(s)-s+0+4Y(s)=\frac{1}{s}-\frac{e^{-\pi s}}{s}$$

$$Y(s)(s^2+4)=\frac{1}{s}-\frac{e^{-\pi s}}{s}+s \Rightarrow Y(s)=\frac{s^2+1-e^{-\pi s}}{s(s^2+4)}$$

$$\frac{s^2+1}{s(s^2+4)}-\frac{e^{-\pi s}}{s(s^2+4)}$$

$$\frac{A}{s}+\frac{Bs+C}{s^2+4}=\frac{As^2+4A+Bs^2+Cs}{s(s^2+4)}$$

$A+B=1 \Rightarrow B=3/4$

$C=0$

$4A=1 \Rightarrow A=1/4$

$$\frac{s^2+1}{s(s^2+4)}=\frac{1}{4}\left(\frac{1}{s}\right)+\frac{3}{4}\left(\frac{s}{s^2+4}\right)$$

$$\frac{1}{s(s^2+4)}=\frac{D}{s}+\frac{Es+F}{s^2+4} \qquad \frac{Ds^2+D4+Es^2+Fs}{s(s^2+4)}$$

$D+E=0$

$4D=1 \Rightarrow D=1/4$

$F=0 \Rightarrow E=-1/4$

$$\frac{e^{-\pi s}}{s(s^2+4)}=\frac{1}{4}\left(\frac{1}{s}\right)e^{-\pi s}-\frac{1}{4}\left(\frac{s}{s^2+4}\right)e^{-\pi s}$$

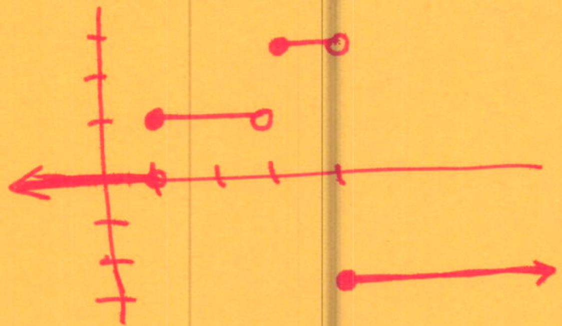
$$y(t)=\frac{1}{4}+\frac{3}{4}\cos 2t-\left(\frac{1}{4}-\frac{1}{4}\cos(2(t-\pi))\right)u_\pi$$

$$\cos(2t-2\pi)=\cos(2t)$$

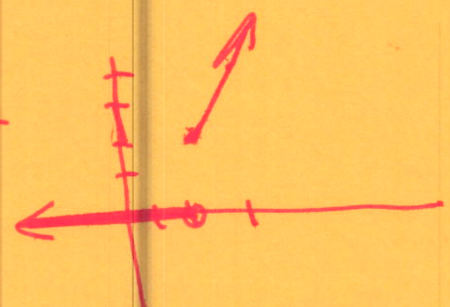
$$y(t)=\begin{cases}\frac{1}{4}+\frac{3}{4}\cos 2t & 0\le t<\pi \\ 1/2\cos 2t & t\ge\pi\end{cases}$$

6a. $\begin{cases} 0 & t<1 \\ 1 & t\geq 1 \end{cases} + 2\begin{cases} 0 & t<3 \\ 1 & t\geq 3 \end{cases} - 6\begin{cases} 0 & t<4 \\ 1 & t\geq 4 \end{cases}$

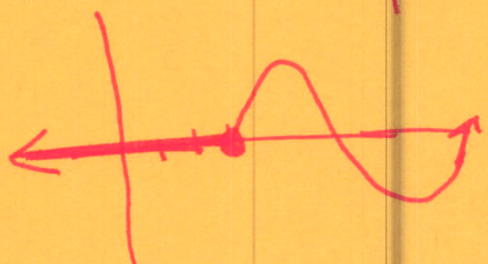
$= \begin{cases} 0 & t<1 \\ 1 & 1\leq t<3 \\ 3 & 3<t<4 \\ -3 & t\geq 4 \end{cases}$



b. $\begin{cases} 0 & t<2 \\ 2(t-1) & t\geq 2 \end{cases} = \begin{cases} 0 & t<2 \\ 2t-2 & t\geq 2 \end{cases}$



c. $\begin{cases} 0 & t<3 \\ \sin(t-3) & t\geq 3 \end{cases}$



7a. $\begin{cases} 0 & 0\leq t<3 & f \\ -2 & 3\leq t<3 & g \\ 2 & 5\leq t<7 & h \\ 1 & t\geq 7 & i \end{cases}$

$g-f = -2-0 = -2$

$h-g = 2-(-2) = 4$

$i-h = 1-2 = -1$

$-2u_3(t) + 4u_5(t) - u_7(t)$ or

$-2u(t-3) + 4u(t-5) - u(t-7)$

b. $\begin{cases} t & 0\leq t<2 & f \\ 2 & 2\leq t<5 & g \\ 7-t & 5\leq t<7 & h \\ 0 & t\geq 7 & i \end{cases}$

$g-f = 2-t = -(t-2) \quad \approx -t$

$h-g = 7-t-2 = 5-t = -(t-5) \quad \approx -t$

$i-h = 0-(7-t) = t-7 \quad \approx t$

$t - (t-2)u_2 - (t-5)u_5 + (t-7)u_7$

8a. $\frac{3x+2}{x^2+x} = \frac{3x+2}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{Ax+A+Bx}{x(x+1)}$

$A+B=3$
$A=2 \Rightarrow B=1$

$= \frac{2}{x} + \frac{1}{x+1}$

b. $\frac{x^2+29x+5}{(x-4)^2(x^2+3)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{Cx+D}{x^2+3}$

$A(x-4)(x^2+3) + B(x^2+3) + (Cx+D)(x-4)^2$

$A(x^3+3x-4x^2-12) + B(x^2+3) + (Cx+D)(x^2-8x+16)$

$Ax^3+3Ax-4Ax^2-12A+Bx^2+3B+Cx^3-C8x^2+16Cx+Dx^2-8Dx+16D$

$A+C=0$
$-4A+B-8C+D=1$
$3A+16C-8D=29$
$-12A+3B+16D=5$

$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ -4 & 1 & -8 & 1 & 1 \\ 3 & 0 & 16 & -8 & 29 \\ -12 & 3 & 0 & 16 & 5 \end{array}\right]$

$A = -\frac{393}{361}$ $B = \frac{137}{19}$
$C = \frac{393}{361}$ $D = \frac{-670}{361}$

$-\frac{393}{361}\left(\frac{1}{x-4}\right) + \frac{137}{19}\left(\frac{1}{(x-4)^2}\right) + \frac{393}{361}\left(\frac{x}{x^2+3}\right) - \frac{670}{361}\left(\frac{1}{x^2+3}\right)$

c. $\frac{3x+11}{x^2-x-6} = \frac{3x+11}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2} = \frac{Ax+2A+Bx-3B}{(x-3)(x+2)}$

$A+B=3$
$2A-3B=11$

$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & -3 & 11 \end{array}\right]$

$A=4$
$B=-1$

$\frac{4}{x-3} - \frac{1}{x+2}$

d. $\frac{x^3+10x^2+3x+36}{(x-1)(x^2+4)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$

$A(x^2+4)^2 + (Bx+C)(x-1)(x^2+4) + (Dx+E)(x-1)$

$A(x^4+8x^2+16) + (Bx+C)(x^3-x^2+4x-4) + Dx^2-Dx+Ex-E$

$Ax^4+8Ax^2+16A+Bx^4-Bx^3+4Bx^2-4Bx+Cx^3-Cx^2+4Cx-4C+Dx^2$
$-Dx+Ex-E$

8d cont'd

$A+B=0$
$-B+C=1$
$8A+4B-C+D=10$
$-4B+4C-D+E=3$
$16A-4C-E=36$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \\ 8 & 4 & -1 & 1 & 0 & 10 \\ 0 & -4 & 4 & -1 & 1 & 3 \\ 16 & 0 & -4 & 0 & -1 & 36 \end{array}\right]$$

$A=2,\ B=-2,\ C=-1,\ D=1,\ E=0$

$$\frac{2}{x-1}-\frac{2x}{x^2+4}-\frac{1}{x^2+4}+\frac{1}{(x^2+4)^2}$$

e. $$\frac{x^2+4}{3x^3+4x^2-4x}=\frac{x^2+4}{x(3x^2+4x-4)}=\frac{x^2+4}{x(3x-2)(x+2)}=\frac{A}{x}+\frac{B}{3x-2}+\frac{C}{x+2}$$

$A(3x^2+4x-4)+Bx(x+2)+Cx(3x-2)$
$3Ax^2+4Ax-4A+Bx^2+2Bx+3Cx^2-2Cx$

$3A+B+3C=1$ $\quad -3+B+3C=1 \Rightarrow B+3C=4$
$4A+2B-2C=0$ $\quad -4+2B-2C=0 \Rightarrow \frac{2B-2C=4}{2}$
$-4A=4 \Rightarrow A=-1$ $\quad B-C=2$

$$\left[\begin{array}{cc|c} 1 & 3 & 4 \\ 1 & -1 & 2 \end{array}\right]$$

$A=-1,\ B=5/2,\ C=1/2$

$$-\frac{1}{x}+\frac{5}{2}\left(\frac{1}{3x-2}\right)+\frac{1}{2}\left(\frac{1}{x+2}\right)$$

f. $$\frac{18}{x^3-3x^2}=\frac{18}{x^2(x-3)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-3}$$

$Ax(x-3)+B(x-3)+Cx^2$
$Ax^2-3Ax+Bx-3B+Cx^2$
$A+C=0$ $\quad C=2$
$-3A+B=0 \Rightarrow -3A-6=0$
$-3B=18$ $\quad -3A=6$
$B=-6$ $\quad A=-2$

$A=-2,\ B=-6,\ C=2$

$$-\frac{2}{x}-\frac{6}{x^2}+\frac{2}{x-3}$$