212 Homework #2 Key

1a. $A\vec{u} = \begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ -1 \end{bmatrix} = \begin{bmatrix} 2\cdot 5 + (-1)(-1) \\ 4\cdot 5 + (-1)(1) \end{bmatrix} = \begin{bmatrix} 10+1 \\ 20-1 \end{bmatrix} = \begin{bmatrix} 11 \\ 19 \end{bmatrix}$

b. $B\vec{v} = \begin{bmatrix} i & 1+i \\ 2-i & -3 \end{bmatrix}\begin{bmatrix} 3i \\ 1-4i \end{bmatrix} = \begin{bmatrix} i(3i) + (1+i)(1-4i) \\ (2-i)(3i) + (-3)(1-4i) \end{bmatrix} = \begin{bmatrix} 3i^2 + 1 - 4i + i - 4i^2 \\ 6i - 3i^2 - 3 + 12i \end{bmatrix} =$

$\begin{bmatrix} -3 + 1 - 3i + 4 \\ 6i + 3 - 3 + 12i \end{bmatrix} = \begin{bmatrix} 2 - 3i \\ 18i \end{bmatrix}$

c. $C\vec{x} = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 1 & 4 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 - 1 - 6 \\ 2 - 1 + 12 \\ -1 + 0 + 3 \end{bmatrix} = \begin{bmatrix} -4 \\ 13 \\ 2 \end{bmatrix}$

d. $AB = \begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} i & 1+i \\ 2-i & -3 \end{bmatrix} = \begin{bmatrix} 2i - 2 + i & 2 + 2i + 3 \\ 4i + 2 - i & 4 + 4i - 3 \end{bmatrix} = \begin{bmatrix} -2 + 3i & 5 + 2i \\ 2 + 3i & 1 + 4i \end{bmatrix}$

e. $BA = \begin{bmatrix} i & 1+i \\ 2-i & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2i + 4 + 4i & -i + 1 + i \\ 4 - 2i - 12 & -2 + i - 3 \end{bmatrix} = \begin{bmatrix} 4 + 6i & 1 \\ -8 - 2i & -5 + i \end{bmatrix}$

f. $B^* = \overline{B}^T = \overline{\begin{bmatrix} i & 1+i \\ 2-i & -3 \end{bmatrix}}^T = \begin{bmatrix} -i & 1-i \\ 2+i & -3 \end{bmatrix}^T = \begin{bmatrix} -i & 2+i \\ 1-i & -3 \end{bmatrix}$

g. $C^T = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 1 & 4 \\ -1 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 1 & 0 \\ -2 & 4 & 1 \end{bmatrix}$

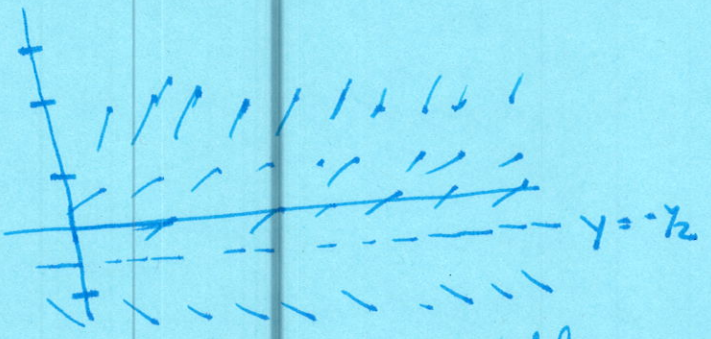
2a. $y' = 1 + 2y = 2(\frac{1}{2} + y)$

$0 = \frac{1}{2} + y$

$y = -\frac{1}{2}$

$y > -\frac{1}{2}$ +

$y < -\frac{1}{2}$ −



unstable

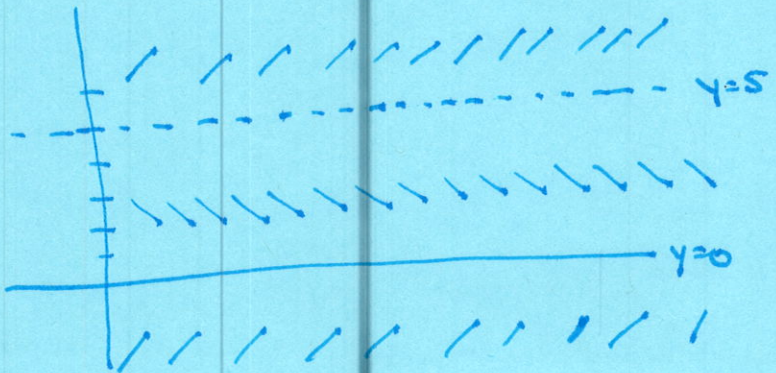
b. $y' = -y(5-y) = y(y-5)$

$0 = -y(5-y)$

$y = 0, y = 5$

$y = 0$ stable

$y = 5$ unstable



H2 Homework #2 Key cont'd

2c. $y' = y(y-2)^2$ $\quad y=0, y=2$

$y=0$ unstable

$y=2$ semi stable

3. See attached

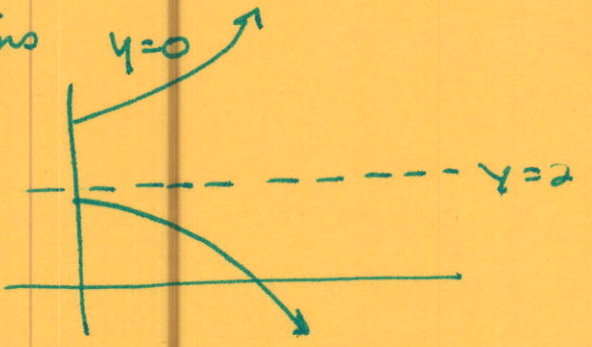
4. a. equilibrium at $y=2$

$y' = y-2$ or $y' = 2-y$

	$y'=y-2$	$y'=2-y$
$y=0$	$-$	$+$

$\boxed{y' = y-2}$ matches signs

Signs



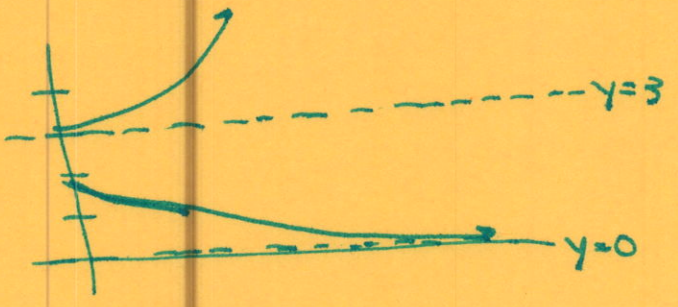
b. equilibrium at $y=0, y=3$

$y' = y(y-3)$ or $y' = -y(y-3)$

	$y'=y(y-3)$	$y'=-y(y-3)$
$y=1$	$-$	$+$
$y=4$	$+$	$-$

$\boxed{y' = y(y-3)}$ matches signs

Signs



5 linear

$\frac{d^3y}{dt^3}$ is y''', $\frac{dy}{dt} = y'$, 3rd y term is just y

all nonlinear components are in t which doesn't count for linearity w/ ODEs

6a. $y' = \frac{x^2}{y(1+x^3)^4} \Rightarrow y\,dy = \frac{x^2}{(1+x^3)^4}\,dx$ $\quad u = 1+x^3$, $du = 3x^2$, $\frac{1}{3}du = x^3$

$\int y\,dy = \int \frac{1}{3}u^{-4}\,du \Rightarrow \frac{1}{2}y^2 = -\frac{1}{9}u^{-3} + C$

$y^2 = -2/9\left(\frac{1}{(1+x^3)^3}\right) = \frac{-2}{9(1+x^3)^3} + C$

66. $y' = \frac{1-2x}{y}$, $y(1) = -2$

$$\int y\,dy = \int 1-2x\,dx \Rightarrow \frac{1}{2}y^2 = x - x^2 + C$$

$$\frac{1}{2}(-2)^2 = 1 - 1^2 + C$$

$$2 = C \qquad \frac{1}{2}y^2 = x - x^2 + 2$$

$$y^2 = 2x - 2x^2 + 4$$

$$y = -\sqrt{2x - 2x^2 + 4} \quad \text{(negative root since } y(1) = -2\text{)}$$

c. $y' + y^2 \sin x = 0$

$y' = -y^2 \sin x$

$$\int \frac{dy}{y^2} = \int -\sin x\,dx \Rightarrow -\frac{1}{y} = \cos x + C \Rightarrow y = \frac{-1}{\cos x + C}$$

d. $xy' = (1-y^2)^{1/2}$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{x} \Rightarrow \arcsin y = \ln x + C$$

e. $\sin 2x\,dx + \cos 3y\,dy = 0 \qquad y(\pi/2) = \pi/3$

$$\int -\sin 2x\,dx = \int \cos 3y\,dy$$

$$\frac{1}{2}\cos 2x + C = \frac{1}{3}\sin 3y$$

$$\frac{1}{2}\cos(2\cdot\pi/2) + C = \frac{1}{3}\sin(3\cdot\pi/3)$$

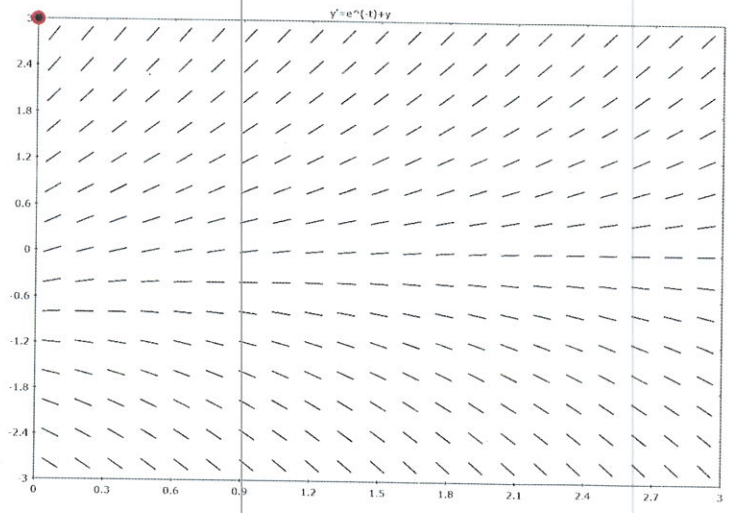
$$\frac{1}{2}\cos\pi + C = \frac{1}{3}\sin\pi$$

$$\frac{1}{2}(-1) + C = 0$$

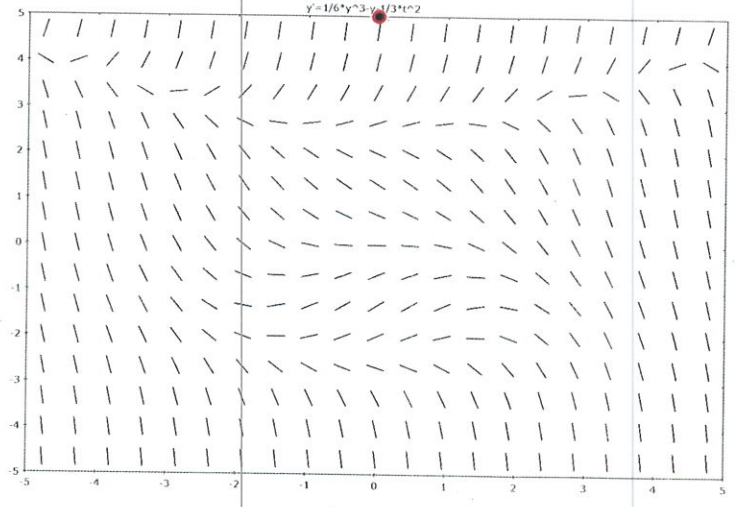
$$C = 1/2$$

$$\frac{1}{2}\cos 2x + 1/2 = \frac{1}{3}\sin 3y$$

3a.



3b.



3c.