212 Homework #4 key

1a. See attached
b. See attached
c. See attached
d. See attached

2a. $y' = 2y - 1 \Rightarrow y' = 2(y - 1/2) \Rightarrow \int \frac{dy}{y-1/2} = \int 2\,dt \Rightarrow \ln(y-1/2) = 2t + C$

$\Rightarrow y - 1/2 = Ae^{2t}$

$y(t) = Ae^{2t} + 1/2$

$1 = Ae^0 + 1/2 \Rightarrow 1/2 = A$

$y(t) = 1/2 e^{2t} + 1/2$

$y(1/2) = 1/2 e^{(1)} + 1/2 \approx 1.8591$

$e^{2t+C} = e^C \cdot e^{2t}$

$A = e^C$

$y(0) = 1$

the solutions are more accurate The more steps are used pretty good estimate

b. $y' = y(3 - ty)$ $y(0) = 2$

$y' = 3y - ty^2 \Rightarrow y' - 3y = -ty^2$ $\quad (1-n)y^{-n} = (1-2)y^{-2} = -1y^{-2}$

$-y^{-2}y' + 3y^{-1} = t \Rightarrow z = y^{-1}$ $\quad z' = -y^{-2}y'$

$z' + 3z = t \Rightarrow \mu = e^{\int 3dt} = e^{3t} \Rightarrow e^{3t}z' + 3e^{3t}z = e^{3t}t \Rightarrow$

$\int (e^{3t}z)' = \int e^{3t}t$ $\quad u = t$ $\quad dv = e^{3t}dt$ $\quad du = dt$ $\quad v = 1/3 e^{3t}$ $\quad \frac{1}{3}te^{3t} - \frac{1}{3}\int e^{3t}dt$

$e^{3t}z = \frac{1}{3}te^{3t} - \frac{1}{9}e^{3t} + C \;) * e^{-3t} \Rightarrow z = \frac{1}{3}t - \frac{1}{9} + Ce^{-3t}$

$\Rightarrow \frac{1}{y} = \frac{1}{3}t - \frac{1}{9} + Ce^{-3t} \Rightarrow \frac{1}{2} = \frac{1}{3}(0) - \frac{1}{9} + Ce^0$

$1/2 + 1/9 = C \Rightarrow C = 7/18$

$\frac{1}{y} = \frac{1}{3}t - \frac{1}{9} + \frac{7}{18}e^{-3t}$ $\quad t = 1/2$ $\quad y(t) = \frac{1}{\frac{1}{3}(1/2) - \frac{1}{9} + \frac{7}{18}e^{-3/2}} \approx 7.026$

nonlinear and kinda off here.

3.a. $6y'' - 5y' + y = 0$ $\quad y(0) = 4,\ y'(0) = 0$

$6r^2 - 5r + 1 = 0$ $\quad (3r-1)(2r-1) = 0$ $\quad r = 1/3,\ r = 1/2$

$y(t) = c_1 e^{1/3 t} + c_2 e^{1/2 t}$ $\quad y' = 1/3 c_1 e^{1/3 t} + 1/2 c_2 e^{1/2 t}$

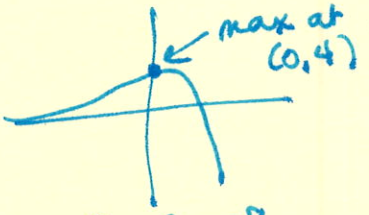
$4 = c_1 + c_2$

$0 = 1/3 c_1 + 1/2 c_2$

$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 1/3 & 1/2 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 12 \\ 0 & 1 & -8 \end{array}\right]$

$c_1 = 12,\ c_2 = -8$

$y(t) = 12e^{1/3 t} - 8e^{1/2 t}$



goes to $-\infty$ as $t \to \infty$

b. $2y'' - 3y' + y = 0$ $\quad y(0) = 2,\ y'(0) = 1/2$

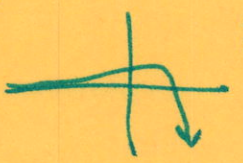
$2r^2 - 3r + 1 = 0$ $\quad (2r-1)(r-1) = 0$ $\quad r = 1/2,\ r = 1$ $\quad y(t) = c_1 e^{1/2 t} + c_2 e^{t}$

3b (cont'd) $y'(t) = \frac{1}{2}c_1 e^{\frac{1}{2}t} + c_2 e^t$

$c_1 + c_2 = 2$
$\frac{1}{2}c_1 + c_2 = \frac{1}{2}$
$\Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ \frac{1}{2} & 1 & \frac{1}{2} \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1 \end{array}\right]$

$c_1 = 3, c_2 = -1$

$y(t) = 3e^{\frac{1}{2}t} - e^t$ max at $\approx (.81, 2.25)$



goes to $-\infty$ as $t \to \infty$

3c. $y'' + 4y' + 5y = 0 \quad y(0) = 1, \; y'(0) = 0$

$r^2 + 4r + 5 = 0$ does not factor $\quad r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$r = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{4}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$

$e^{(-2+i)t} = e^{-2t} \cdot e^{it} = e^{-2t}(\cos t + i \sin t) \Rightarrow$

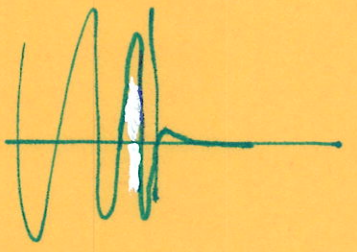
$y(t) = c_1 e^{-2t}\cos t + c_2 e^{-2t}\sin t \qquad y' = -2c_1 e^{-2t}\cos t - c_1 e^{-2t}\sin t - 2c_2 e^{-2t}\sin t + c_2 e^{-2t}\cos t$

$c_1(1) + c_2(0) = 1 \Rightarrow c_1 = 1$

$-2(1)(1)(1) - (1)(1)(0) - 2c_2(1)(0) + c_2(1)(1) = 0$

$-2 + c_2 = 0 \Rightarrow c_2 = 2$

$y(t) = e^{-2t}\cos t + 2e^{-2t}\sin t$



infinite # of critical points $\to 0$ as $t \to \infty$

3d. $y'' + 4y' + 4y = 0 \quad y(-1) = 2 \quad y'(-1) = 1$

$r^2 + 4r + 4 = 0 \quad (r+2) = 0 \quad r = -2 \qquad y(t) = c_1 e^{-2t} + c_2 t e^{-2t}$

$y'(t) = -2c_1 e^{-2t} + c_2 e^{-2t} - 2c_2 t e^{-2t}$

$c_1 e^{+2} + c_2(-1)e^2 = 2 \Rightarrow c_1 - c_2 = \frac{2}{e^2} \Rightarrow 2c_1 - 2c_2 = \frac{4}{e^2}$

$-2c_1 e^2 + c_2 e^2 - 2c_2(-1)e^2 = 1 \quad -2c_1 + 3c_2 = \frac{1}{e^2} \quad -2c_1 + 3c_2 = \frac{1}{e^2}$

$c_2 = \frac{5}{e^2}$

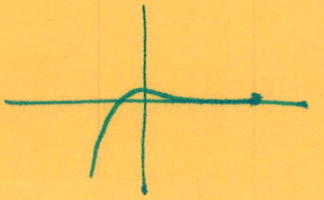
$c_1 = \frac{2}{e^2} - \frac{5}{e^2} = \frac{3}{e^2}$

$y(t) = \frac{3}{e^2}e^{-2t} + \frac{5}{e^2}te^{-2t} = 3e^{-2t-2} + 5te^{-2t-2}$

$= 3e^{-2(t+1)} + 5te^{-2(t+1)}$

as $t \to \infty$, $y \to 0$

max at $\approx (-0.1, .4132)$



4. $|A| = 2(1) - 4(-1) = 2+4 = 6$

$|B| = i(-3) - (2-i)(1+i) = -3i - (2+2i-i+1) = -3i - 3 - i = -4i - 3$

$-3-4i$

$|C| = 3\begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & 4 \\ -1 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} = 3(1-0) - 1(2+4) - 2(0+1) =$

$3 - 6 - 2 = -5$

5. a. $x^2+3x-4=0 \Rightarrow (x+4)(x-1)=0$ $x=-4, x=1$ 2 solutions

b. $18x^3 - 63x^2 + 9x = 0 \Rightarrow 9x(2x^2 - 7x + 1) = 0$ $x=0$

$x = \frac{7 \pm \sqrt{49-8}}{4} = \frac{7}{4} \pm \frac{\sqrt{41}}{4}$ 3 solutions

c. $125 - 8y^3 = 0$ $(5-2y)(25+10y+4y^2)=0$ $y = \frac{5}{2}$ 3 solutions

$y = \frac{-10 \pm \sqrt{100-4(25)(4)}}{8} = \frac{-10 \pm \sqrt{100-400}}{8} = \frac{-10 \pm 10\sqrt{3}i}{8} = -\frac{5}{4} \pm \frac{5\sqrt{3}}{4}i$

d. $64r^3 + 27 = 0$ $(4r+3)(16r^2 - 12r + 9) = 0$ $r = 3/4$ 3 solutions

$r = \frac{12 \pm \sqrt{144 - 4 \cdot 16 \cdot 9}}{32} = \frac{12 \pm \sqrt{144-576}}{32} = \frac{12 \pm \sqrt{-432}}{32}$ $\frac{12 \pm 12\sqrt{3}i}{32}$

$= \frac{3}{8} \pm \frac{3}{8}\sqrt{3}i$

e. $2m^3 - 18m = 0 \Rightarrow 2m(m^2-9)=0$ $m=0$ $m=\pm 3$ 3 solutions

f. $2x^2 - 98 = 0 \Rightarrow 2(x^2-49)=0$ $x=\pm 7$ 2 solutions

g. $3r^2 - 75 = 0 \Rightarrow 3(r^2-25)=0$ $r=\pm 5$ 2 solutions

h. $x^2+11x+24=0 \Rightarrow (x+3)(x+8)=0$ $x=-3,-8$ 2 solutions

i. $14t^2 - 9t + 1 = 0 \Rightarrow (2t-1)(7t-1)=0$ $t = 1/2, 1/7$ 2 solutions

j. $n^3 - 28 + 7n^2 - 4n = 0 \Rightarrow n^3 + 7n^2 - 4n - 28 = 0 \Rightarrow n^2(n+7) - 4(n+7) = 0$

$(n+7)(n^2-4)=0$ $n=-7, \pm 2$ 3 solutions

k. $20 - 3x - 2x^2 = 0 \Rightarrow 2x^2 + 3x - 20 = 0$ $(2x-5)(x+4)=0$

$x = 5/2, -4$ 2 solutions

l. $16a^2 - 56a + 49 = 0 \Rightarrow (4a-7)^2 = 0$ $a = 7/4$ (repeated) $\Rightarrow$ 1 solutions

m. $12x^2 + 34x + 24 = 0 \Rightarrow 2(6x^2 + 17x + 12) = 0$ $(2x+3)(3x+4)=0$

$x = -3/2, -4/3$ 2 solutions

n. $x^4 - 14x^2 - 32 = 0$ $(x^2-16)(x^2+2)=0$ $x = \pm 4$, $x = \sqrt{2}i, -\sqrt{2}i$ 4 solutions

o. $s^6 - 1 = 0 \Rightarrow (s^3-1)(s^3+1)=0 \Rightarrow (s-1)(s^2+s+1)(s+1)(s^2-s+1)=0$ 6 solutions

$s = \pm 1, \pm 1/2 \pm \frac{\sqrt{3}}{2}i$ $\frac{-1 \pm \sqrt{1-4}}{2}$ $\frac{1 \pm \sqrt{1-4}}{2}$

2/2 Homework #4 Key

n	t_n	y_n	delta_t	m_n=2y_n-1	y_n+1=y_n+delta_t*m_n
0	1	0	0.5	-1	-0.5
1	1.5	-0.5	0.5	-2	-1.5
2	**2**	**-1.5**	0.5	-4	-3.5

1a: (2,-1.5)

n	t_n	y_n	delta_t	m_n=5-3*sqrt(y_n)	y_n+1=y_n+delta_t*m_n
0	1	4	0.1	-1	3.9
1	1.1	3.9	0.1	-0.924525297	3.80754747
2	1.2	3.807547	0.1	-0.853881382	3.722159332
3	1.3	3.722159	0.1	-0.787869555	3.643372377
4	1.4	3.643372	0.1	-0.726286003	3.570743776
5	**1.5**	**3.570744**	0.1	-0.66892353	3.503851423

1b: (1.5,3.57)

n	t_n	y_n	delta_t	m_n=y_n(2-t_n*y_n)	y_n+1=y_n+delta_t*m_n
0	2	1	0.333333	0	1
1	2.333333	1	0.333333	-0.333333333	0.888888889
2	2.666667	0.888889	0.333333	-0.329218107	0.77914952
3	**3**	**0.77915**	0.333333	-0.262922883	0.691508559

1c: (3,0.78)

n	t_n	y_n	delta_t	m_n=3*t_n^2/(y_n^2	y_n+1=y_n+delta_t*m_n
0	3	1	0.2	-9	-0.8
1	3.2	-0.8	0.2	-9.142857143	-2.628571429
2	3.4	-2.62857	0.2	11.92003367	-0.244564695
3	3.6	-0.24456	0.2	-9.867549191	-2.218074533
4	3.8	-2.21807	0.2	47.09439779	7.200805025
5	4	7.200805	0.2	1.003101401	7.401425305
6	4.2	7.401425	0.2	1.042120072	7.609849319
7	4.4	7.609849	0.2	1.077355004	7.82532032
8	4.6	7.82532	0.2	1.109099192	8.047140159
9	4.8	8.04714	0.2	1.137656714	8.274671501
10	5	8.274672	0.2	1.163328382	8.507337178
11	5.2	8.507337	0.2	1.186402253	8.744617628
12	5.4	8.744618	0.2	1.207147881	8.986047205
13	5.6	8.986047	0.2	1.225813309	9.231209866
14	5.8	9.23121	0.2	1.242623989	9.479734664

15	6	9.479735	0.2	1.257782979	9.73129126
16	6.2	9.731291	0.2	1.271471944	9.985585649
17	6.4	9.985586	0.2	1.283852618	10.24235617
18	6.6	10.24236	0.2	1.295068493	10.50136987
19	6.8	10.50137	0.2	1.30524658	10.76241919
20	**7**	**10.76242**	0.2	1.314499133	11.02531901

1d: (7,10.76)

2a.

n	t_n	y_n	delta_t	m_n=2y_n-1	y_n+1=y_n+delta_t*m_n
0	0	1	0.1	1	1.1
1	0.1	1.1	0.1	1.2	1.22
2	0.2	1.22	0.1	1.44	1.364
3	0.3	1.364	0.1	1.728	1.5368
4	0.4	1.5368	0.1	2.0736	1.74416
5	**0.5**	**1.74416**	0.1	2.48832	1.992992

(0.5,1.7442)

n	t_n	y_n	delta_t	m_n=2y_n-1	y_n+1=y_n+delta_t*m_n
0	0	1	0.05	1	1.05
1	0.05	1.05	0.05	1.1	1.105
2	0.1	1.105	0.05	1.21	1.1655
3	0.15	1.1655	0.05	1.331	1.23205
4	0.2	1.23205	0.05	1.4641	1.305255
5	0.25	1.305255	0.05	1.61051	1.3857805
6	0.3	1.385781	0.05	1.771561	1.47435855
7	0.35	1.474359	0.05	1.9487171	1.571794405
8	0.4	1.571794	0.05	2.14358881	1.678973846
9	0.45	1.678974	0.05	2.357947691	1.79687123
10	**0.5**	**1.796871**	0.05	2.59374246	1.926558353

(0.5,1.7969)

n	t_n	y_n	delta_t	m_n=2y_n-1	y_n+1=y_n+delta_t*m_n
0	0	1	0.01	1	1.01
1	0.01	1.01	0.01	1.02	1.0202
2	0.02	1.0202	0.01	1.0404	1.030604
3	0.03	1.030604	0.01	1.061208	1.04121608
4	0.04	1.041216	0.01	1.08243216	1.052040402
5	0.05	1.05204	0.01	1.104080803	1.06308121
6	0.06	1.063081	0.01	1.126162419	1.074342834
7	0.07	1.074343	0.01	1.148685668	1.085829691
8	0.08	1.08583	0.01	1.171659381	1.097546284
9	0.09	1.097546	0.01	1.195092569	1.10949721
10	0.1	1.109497	0.01	1.21899442	1.121687154
11	0.11	1.121687	0.01	1.243374308	1.134120897
12	0.12	1.134121	0.01	1.268241795	1.146803315
13	0.13	1.146803	0.01	1.29360663	1.159739382
14	0.14	1.159739	0.01	1.319478763	1.172934169
15	0.15	1.172934	0.01	1.345868338	1.186392853
16	0.16	1.186393	0.01	1.372785705	1.20012071
17	0.17	1.200121	0.01	1.400241419	1.214123124
18	0.18	1.214123	0.01	1.428246248	1.228405586
19	0.19	1.228406	0.01	1.456811173	1.242973698

20	0.2	1.242974	0.01	1.485947396	1.257833172
21	0.21	1.257833	0.01	1.515666344	1.272989835
22	0.22	1.27299	0.01	1.545979671	1.288449632
23	0.23	1.28845	0.01	1.576899264	1.304218625
24	0.24	1.304219	0.01	1.608437249	1.320302997
25	0.25	1.320303	0.01	1.640605994	1.336709057
26	0.26	1.336709	0.01	1.673418114	1.353443238
27	0.27	1.353443	0.01	1.706886477	1.370512103
28	0.28	1.370512	0.01	1.741024206	1.387922345
29	0.29	1.387922	0.01	1.77584469	1.405680792
30	0.3	1.405681	0.01	1.811361584	1.423794408
31	0.31	1.423794	0.01	1.847588816	1.442270296
32	0.32	1.44227	0.01	1.884540592	1.461115702
33	0.33	1.461116	0.01	1.922231404	1.480338016
34	0.34	1.480338	0.01	1.960676032	1.499944776
35	0.35	1.499945	0.01	1.999889553	1.519943672
36	0.36	1.519944	0.01	2.039887344	1.540342545
37	0.37	1.540343	0.01	2.080685091	1.561149396
38	0.38	1.561149	0.01	2.122298792	1.582372384
39	0.39	1.582372	0.01	2.164744768	1.604019832
40	0.4	1.60402	0.01	2.208039664	1.626100228
41	0.41	1.6261	0.01	2.252200457	1.648622233
42	0.42	1.648622	0.01	2.297244466	1.671594678
43	0.43	1.671595	0.01	2.343189355	1.695026571
44	0.44	1.695027	0.01	2.390053142	1.718927103
45	0.45	1.718927	0.01	2.437854205	1.743305645
46	0.46	1.743306	0.01	2.486611289	1.768171758
47	0.47	1.768172	0.01	2.536343515	1.793535193
48	0.48	1.793535	0.01	2.587070385	1.819405897
49	0.49	1.819406	0.01	2.638811793	1.845794015
50	**0.5**	**1.845794**	0.01	2.691588029	1.872709895

(0.5,1.8458)

26.

n	t_n	y_n	delta_t	m_n=y_n(3-t_n*y_n)	y_n+1=y_n+delta_t*m_n
0	0	2	0.1	6	2.6
1	0.1	2.6	0.1	7.124	3.3124
2	0.2	3.3124	0.1	7.742801248	4.086680125
3	0.3	4.08668	0.1	7.249754042	4.811655529
4	0.4	4.811656	0.1	5.174155015	5.32907103
5	**0.5**	**5.329071**	0.1	1.787714067	5.507842437

(0.5,5.3291)

n	t_n	y_n	delta_t	m_n=y_n(3-t_n*y_n)	y_n+1=y_n+delta_t*m_n
0	0	2	0.05	6	2.3
1	0.05	2.3	0.05	6.6355	2.631775
2	0.1	2.631775	0.05	7.202701035	2.991910052
3	0.15	2.99191	0.05	7.633001292	3.373560116
4	0.2	3.37356	0.05	7.844498777	3.765785055
5	0.25	3.765785	0.05	7.752070895	4.1533886
6	0.3	4.153389	0.05	7.284974741	4.517637337
7	0.35	4.517637	0.05	6.409745523	4.838124613
8	0.4	4.838125	0.05	5.151393931	5.09569431
9	0.45	5.095694	0.05	3.602337705	5.275811195
10	0.5	5.275811	0.05	1.910341703	5.37132828

(0.5,5.2758)

n	t_n	y_n	delta_t	m_n=y_n(3-t_n*y_n)	y_n+1=y_n+delta_t*m_n
0	0	2	0.01	6	2.06
1	0.01	2.06	0.01	6.137564	2.12137564
2	0.02	2.121376	0.01	6.274122228	2.184116862
3	0.03	2.184117	0.01	6.409239593	2.248209258
4	0.04	2.248209	0.01	6.54244998	2.313633758
5	0.05	2.313634	0.01	6.673256216	2.38036632
6	0.06	2.380366	0.01	6.801130331	2.448377623
7	0.07	2.448378	0.01	6.925514161	2.517632765
8	0.08	2.517633	0.01	7.045820316	2.588090968
9	0.09	2.588091	0.01	7.161433567	2.659705304
10	0.1	2.659705	0.01	7.271712681	2.732422431
11	0.11	2.732422	0.01	7.375992735	2.806182358
12	0.12	2.806182	0.01	7.473587943	2.880918238
13	0.13	2.880918	0.01	7.563795027	2.956556188
14	0.14	2.956556	0.01	7.645897135	3.033015159
15	0.15	3.033015	0.01	7.719168334	3.110206842
16	0.16	3.110207	0.01	7.782878671	3.188035629
17	0.17	3.188036	0.01	7.836299788	3.266398627
18	0.18	3.266399	0.01	7.878711083	3.345185738
19	0.19	3.345186	0.01	7.909406366	3.424279802
20	0.2	3.42428	0.01	7.927700973	3.503556811
21	0.21	3.503557	0.01	7.932939265	3.582886204
22	0.22	3.582886	0.01	7.924502431	3.662131228
23	0.23	3.662131	0.01	7.901816504	3.741149393
24	0.24	3.741149	0.01	7.864360472	3.819792998
25	0.25	3.819793	0.01	7.811674357	3.897909742
26	0.26	3.89791	0.01	7.743367133	3.975343413

27	0.27	3.975343	0.01	7.659124321	4.051934656
28	0.28	4.051935	0.01	7.55871512	4.127521807
29	0.29	4.127522	0.01	7.441998904	4.201941796
30	0.3	4.201942	0.01	7.308930931	4.275031106
31	0.31	4.275031	0.01	7.159567121	4.346626777
32	0.32	4.346627	0.01	6.994067743	4.416567454
33	0.33	4.416567	0.01	6.812699897	4.484694453
34	0.34	4.484694	0.01	6.615838684	4.55085284
35	0.35	4.550853	0.01	6.40396697	4.61489251
36	0.36	4.614893	0.01	6.177673694	4.676669247
37	0.37	4.676669	0.01	5.9376507	4.736045754
38	0.38	4.736046	0.01	5.684688096	4.792892635
39	0.39	4.792893	0.01	5.419668179	4.847089316
40	0.4	4.847089	0.01	5.143558013	4.898524897
41	0.41	4.898525	0.01	4.857400763	4.947098904
42	0.42	4.947099	0.01	4.562305934	4.992721964
43	0.43	4.992722	0.01	4.25943867	5.03531635
44	0.44	5.035316	0.01	3.950008322	5.074816434
45	0.45	5.074816	0.01	3.635256475	5.111168998
46	0.46	5.111169	0.01	3.316444672	5.144333445
47	0.47	5.144333	0.01	2.994842036	5.174281865
48	0.48	5.174282	0.01	2.671713041	5.200998996
49	0.49	5.200999	0.01	2.348305616	5.224482052
50	**0.5**	**5.224482**	0.01	2.0258398	5.24474045

(0.5,5.2245)