

## 212 Lab #4 Key

My comments are in blue. You will need to select initial conditions, or choose an arbitrary value of the unknown constant to obtain an example graph. For these problem, an arbitrary example is fine.

syms x y t c

1a.

```
>> f=dsolve('Dy=exp(2*x)+y-1','x')
```

f =

$$2*\exp(x)*\cosh(x) + C4*\exp(x)$$

```
>> ezplot(2*exp(x)*cosh(x) + exp(x),[0,2])
```

1b.

```
>> g=dsolve('Dy=(y/x+6*x)/(2-ln(x))','x')
```

g =

$$C6/(\log(x) - 2) - (3*x^2)/(\log(x) - 2)$$

```
>> ezplot(1/(log(x) - 2) - (3*x^2)/(log(x) - 2),[0.1,3])
```

Log is not defined at zero.

1c.

```
>> h=dsolve('Dy=(y^2+1)/y*(cos(x))^2','x')
```

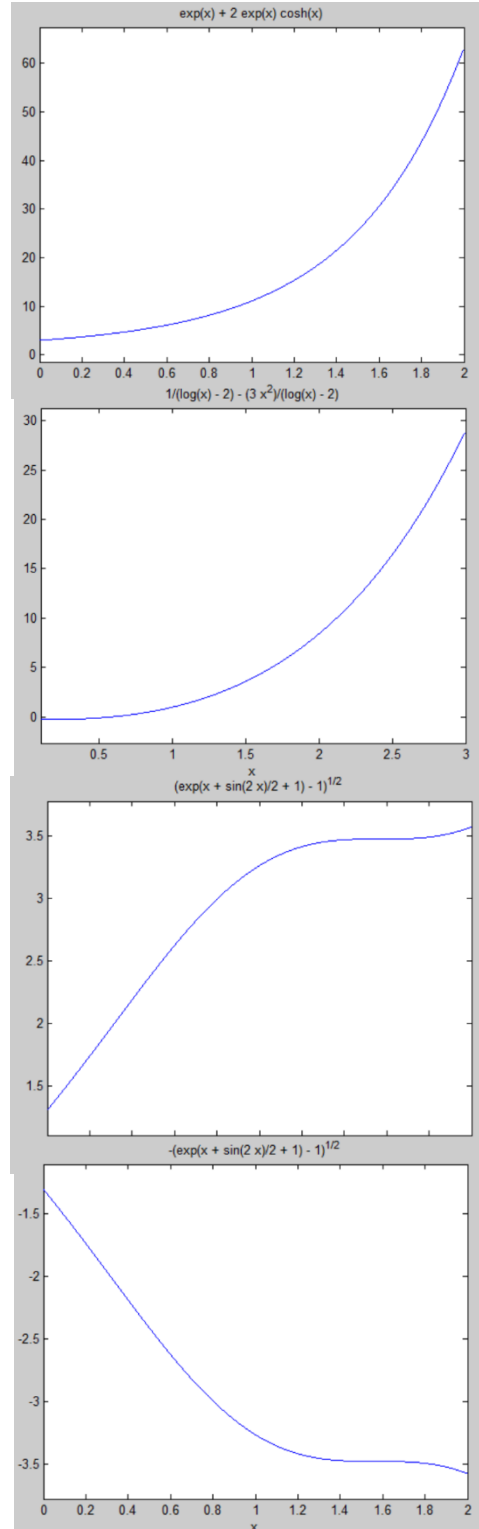
h =

$$\begin{aligned} &(\exp(C9 + x + \sin(2*x)/2) - 1)^{1/2} \\ &-(\exp(C9 + x + \sin(2*x)/2) - 1)^{1/2} \\ &\quad i \\ &\quad -i \end{aligned}$$

```
>> ezplot((exp(1 + x + sin(2*x)/2) - 1)^(1/2),[0,2])
```

```
>> ezplot(-(exp(1 + x + sin(2*x)/2) - 1)^(1/2),[0,2])
```

This equation has two solutions depending on whether y is initially positive or negative, so MatLab lists all the possibilities (including two purely imaginary solutions which we ignore).



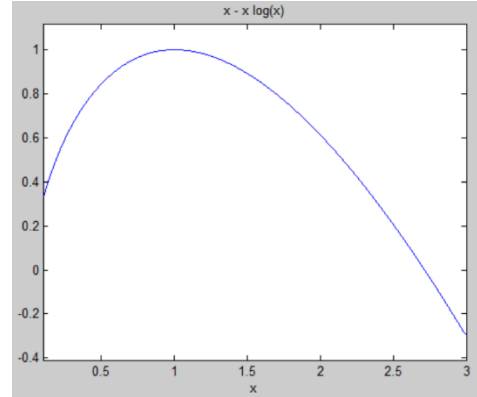
1d.

```
>> j=dsolve('Dy=(y-x)/x','x')
```

j =

$C12*x - x*\log(x)$

```
>> ezplot(x - x*log(x),[0.1,3])
```



1e.

```
>> k=dsolve('Dy=y^2*(y-1)*(y-5)','t')
```

Warning: Explicit solution could not be found; implicit solution returned.

> In dsolve at 201

k =

5  
1  
0

```
solve(log((y^24*(y - 5))/(y - 1)^25) - 20/y == 100*C15 + 100*t, y)
```

```
>> k=dsolve('Dy=y^2*(y-1)*(y-5)','y(0)=0.1','t')
```

Warning: Explicit solution could not be found; implicit solution returned.

> In dsolve at 201

k =

```
solve(log((y^24*(y - 5))/(y - 1)^25) - 20/y == 100*t + log(49/717897987691852588770249) - 200, y)
```

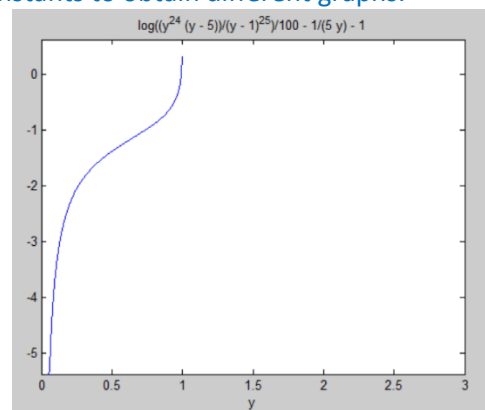
In order to graph this, in addition to substituting in the variable, solve the expression for t instead (replace 'y' at the end of the expression with 't'). The graph you produce will have the axes flipped, but you will be able to obtain a graph. Select other values of the constants to obtain different graphs.

```
>> solve(log((y^24*(y - 5))/(y - 1)^25) - 20/y == 100 + 100*t, t)
```

ans =

```
log((y^24*(y - 5))/(y - 1)^25)/100 - 1/(5*y) - 1
```

```
>> ezplot(ans,[0,3])
```



Above I chose the constant to be 1. Below, I chose it to be 3, and then extended the range of the graph to capture additional y-values on all sides of the three equilibria.

```
>> solve(log((y^24*(y - 5))/(y - 1)^25) - 20/y == 100*3 + 100*t, t)
```

ans =

$\log((y^{24}(y-5))/(y-1)^{25})/100 - 1/(5*y) - 3$

>> ezplot(ans,[-2,7])

1f.

>> l= dsolve('Dy+4\*t\*y/(1+t^2)=(1+t^2)^(-3)', 't')

l =

$C22/(t^2 + 1)^2 + \text{atan}(t)/(t^2 + 1)^2$

>> ezplot(2/(t^2 + 1)^2 + atan(t)/(t^2 + 1)^2,[-pi/2,pi/2])

1g.

>> m=dsolve('y\*Dy+1/x\*y^2=x\*sqrt(y)', 'x')

m =

$((C25 + 3*x^{7/2})/(7*x^{3/2}))^{2/3}$

>> ezplot(((5 + 3\*x^{7/2})/(7\*x^{3/2}))^{2/3},[0,4])

1h.

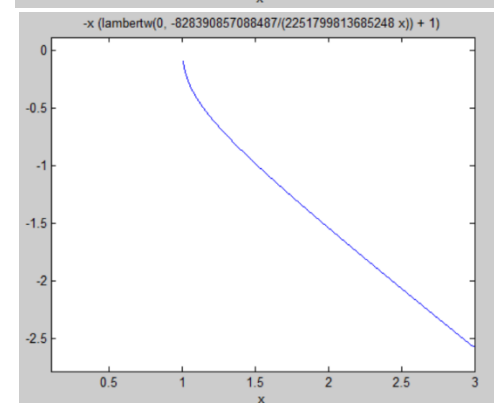
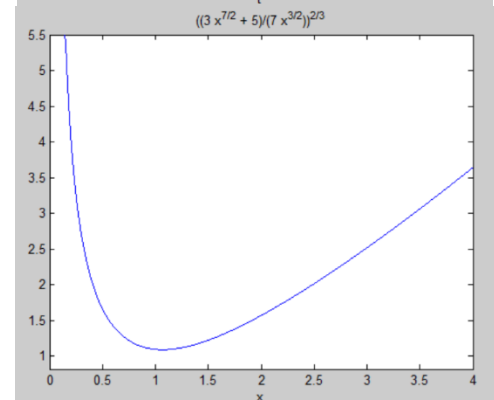
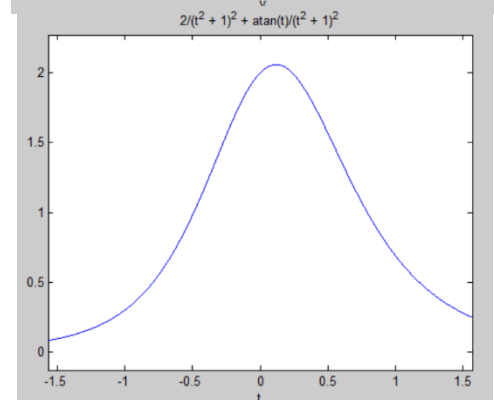
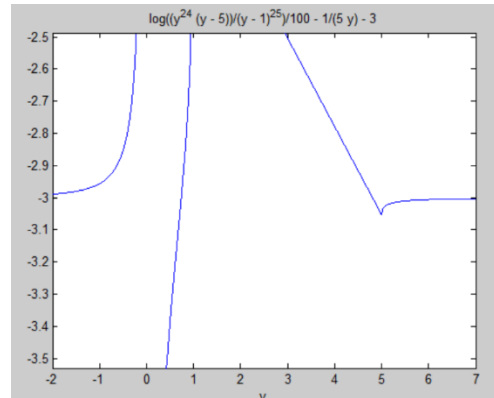
>> n=dsolve('Dy=x/y+y/x+1', 'x')

n =

$$-x \cdot (\text{lambertw}(0, -(C29 \cdot \exp(-1))/x) + 1)$$

This expression produces an integral that can't be evaluated explicitly, but can be graphed numerically. Note that the original equation is not defined at x=0 when you graph it.

>> ezplot(-x\*(lambertw(0, -(exp(-1))/x) + 1),[0.1,3])



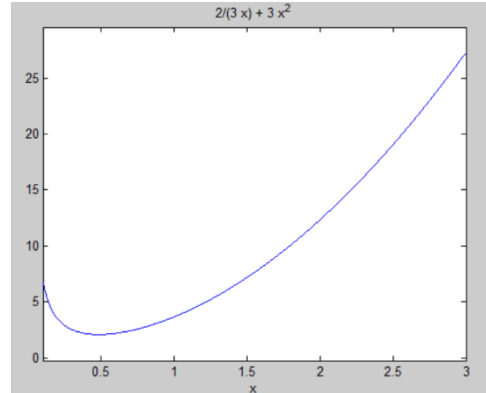
1i.

```
>> o=dsolve('x^2*D2y-2*y=0','x')
```

o =

$C32/(3*x) + C33*x^2$

```
>> ezplot(2/(3*x) + 3*x^2,[0.1,3])
```



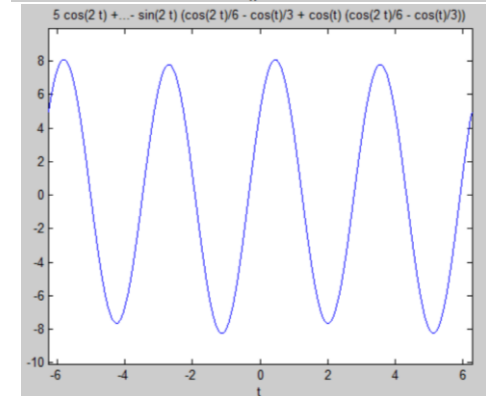
1j.

```
>> p=dsolve('D2y+4*y=sin(t)','t')
```

p =

$\sin(5*t)/24 - \sin(3*t)/8 + \sin(t)/6 - \sin(2*t)*(\cos(2*t)/6 - \cos(t)/3 + \cos(t)*(\cos(2*t)/6 - \cos(t)/3)) + C35*\cos(2*t) + C36*\sin(2*t)$

```
>> ezplot(sin(5*t)/24 - sin(3*t)/8 + sin(t)/6 -  
sin(2*t)*(cos(2*t)/6 - cos(t)/3 + cos(t)*(cos(2*t)/6 - cos(t)/3)) +  
5*cos(2*t) + 6*sin(2*t),[-2*pi,2*pi])
```



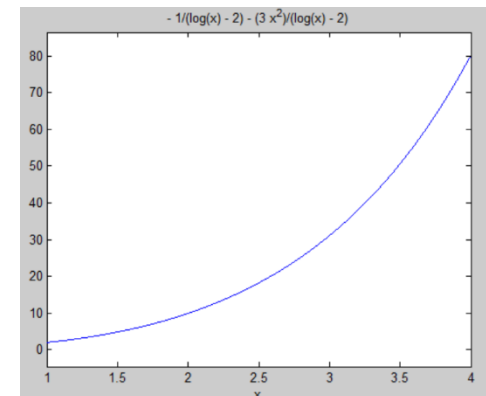
2.

```
>> g=dsolve('Dy=(y/x+6*x)/(2-ln(x))','y(1)=2','x')
```

g =

$-1/(\log(x) - 2) - (3*x^2)/(\log(x) - 2)$

```
>> ezplot(g,[1,4])
```



3.

```
>> figure, hold on
```

```
>>
```

```
>> kt=solve(log((y^24*(y-5))/(y-1)^25) - 20/y == 100*t +  
log((c^24*(c-5))/(c-1)^25) - 20/c, t)
```

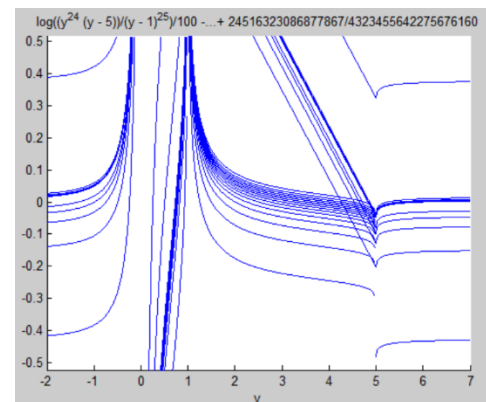
kt =

$\log((y^{24}*(y-5))/(y-1)^{25})/100 - \log((c^{24}*(c-5))/(c-1)^{25})/100 + 1/(5*c) - 1/(5*y)$

```
>>
```

```
>> for cval=-1:0.25:6
```

```
ezplot(subs(kt,c,cval),[-2,7]),end
```



4.

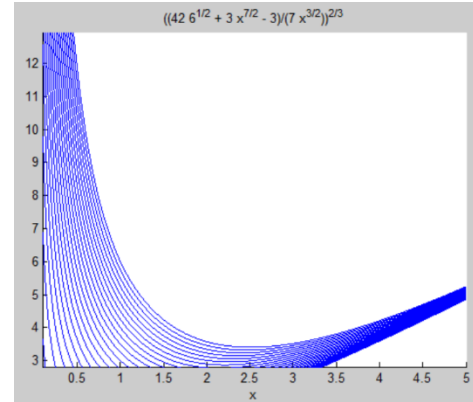
Answers will vary here. I selected 1g. For this problem, since  $c$  appears under a radical, values must be between  $[0,6]$  rather than  $[-1,6]$ . Such complications will differ from problem to problem.

```
>> figure, hold on
>> m=dsolve('y*Dy+1/x*y^2=x*sqrt(y)', 'y(1)=c', 'x')
```

m =

$$\left(\frac{7c^{3/2} + 3x^{7/2} - 3}{7x^{3/2}}\right)^{2/3}$$

```
>> for cval=0:0.25:6
ezplot(subs(m,c,cval),[0.1,5]),end
>> hold off
```



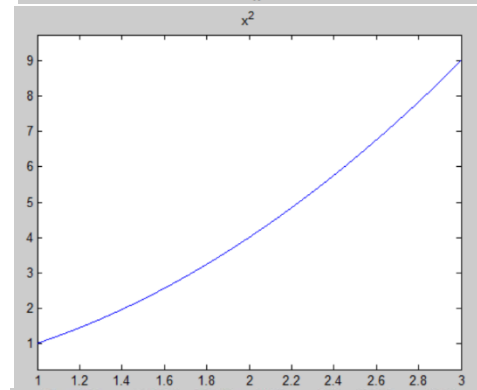
5.

```
>> o=dsolve('x^2*D2y-2*y=0', 'y(1)=1', 'Dy(1)=2', 'x')
```

o =

$$x^2$$

```
>> ezplot(o,[1,3])
```



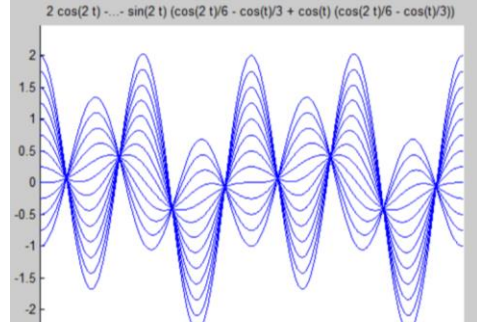
6.

```
>> p=dsolve('D2y+4*y=sin(t)', 'y(0)=c', 'Dy(0)=0', 't')
```

p =

$$\sin(5t)/24 - \sin(3t)/8 - \sin(2t)/3 + \sin(t)/6 - \sin(2t) \cdot (\cos(2t)/6 - \cos(t)/3) + \cos(t) \cdot (\cos(2t)/6 - \cos(t)/3) + c \cdot \cos(2t)$$

```
>> figure, hold on
>> for cval=-1:0.25:2
ezplot(subs(p,c,cval),[-2*pi,2*pi]),end
>> hold off
```



7.

```
>> p=dsolve('D2y+4*y=sin(t)', 'y(0)=0', 'Dy(0)=c', 't')
```

p =

$$\sin(5t)/24 - \sin(3t)/8 + \sin(t)/6 - \sin(2t) \cdot (\cos(2t)/6 - \cos(t)/3) + \cos(t) \cdot (\cos(2t)/6 - \cos(t)/3) + \sin(2t) \cdot (c/2 - 1/3)$$

```
>> figure, hold on
>> for cval=-1:0.25:2
ezplot(subs(p,c,cval),[-2*pi,2*pi]),end
>> hold off
```

