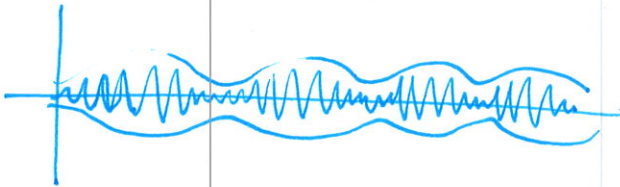


Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Under what conditions do beats occur? Sketch a graph of a beat phenomenon.

beats occur in an undamped system where the forcing frequency is similar but not identical to the natural frequency (and is periodic).



2. If $y(t) = 3e^{-t} \cos 2t - 2e^{-t} \sin 2t + \cos 3t$ for a forced spring system, determine:

- i. The transient solution

$$3e^{-t} \cos 2t - 2e^{-t} \sin 2t$$

- ii. The steady-state solution

$$\cos 3t$$

- iii. Does the system experience beats or resonance? Why or why not?

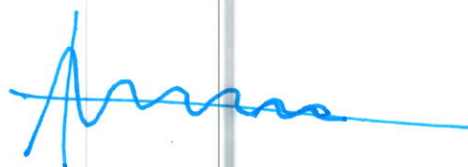
neither; This system is damped

- iv. What is the natural frequency/quasi-frequency?

quasi-frequency (of transient system) is $2 = \omega$

- v. Sketch the graph. How many times does it cross equilibrium?

infinitely many times



3. Find the inverse of $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix} = A$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{6 - 4} \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} 1 & 1/2 \\ -2 & 3/2 \end{bmatrix}$$