

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Solve the third order ODE $8y''' - 125y = 0$. Write the general solution.

$$8r^3 - 125 = 0$$

$$(2r - 5)(4r^2 + 10r + 25) = 0$$

$$r = \frac{5}{2} \quad r = \frac{-10 \pm \sqrt{100 - 10 \cdot 25}}{8} = \frac{-10 \pm 10\sqrt{3}i}{8} = -\frac{5}{4} \pm \frac{5\sqrt{3}i}{4}$$

$$y(t) = c_1 e^{\frac{5}{2}t} + c_2 e^{-\frac{5}{4}t} \cos\left(\frac{5\sqrt{3}}{4}t\right) + c_3 e^{-\frac{5}{4}t} \sin\left(\frac{5\sqrt{3}}{4}t\right)$$

2. Use Cramer's Rule to solve the system $\begin{cases} x_1 + x_2 = 2 \\ x_1 + 2x_2 = 1 \end{cases}$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \quad \det A = 2 - 1 = 1$$

$$A_1 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad \det A_1 = 4 - 1 = 3$$

$$A_2 = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \quad \det A_2 = 1 - 2 = -1$$

$$x_1 = \frac{\det A_1}{\det A} = \frac{3}{1} = 3$$

$$x_2 = \frac{\det A_2}{\det A} = \frac{-1}{1} = -1$$