MAT 212, Quiz #17, Summer 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use the definition $\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$ to find $\mathscr{L}\{t^2\}$.

$$\int_0^\infty t^2 e^{-st} dt =$$

+/-	u	dv
+	t^2	e^{-st}
−	$2t$	$-\frac{1}{s}e^{-st}$
+	2	$\frac{1}{s^2}e^{-st}$
−	0	$-\frac{1}{s^3}e^{-st}$

$$-\frac{t^2}{s}e^{-st} - \frac{2t}{s^2}e^{-st} - \frac{2}{s^3}e^{-st}\Big|_0^\infty$$

$$-0+0-0+0-0+\frac{2}{s^3}(1)$$

$$= \frac{2}{s^3}$$

2. Determine if the functions $f(t) = 2t+1, g(t) = t^3 - 1$ are orthogonal under the inner product $\langle f, g\rangle = \int_{-1}^1 f(t)g(t)dt$.

$$\int_{-1}^1 (2t+1)(t^3-1)\,dt = \int_{-1}^1 2t^4 - 2t + t^3 - 1\,dt =$$

$$\int_{-1}^1 2t^4 - 1\,dt + \underbrace{\int_{-1}^1 -2t + t^3 dt}_{\text{odd} = 0} = \int_{-1}^1 \underbrace{2t^4 - 1}_{\text{even}}\,dt =$$

$$2\int_0^1 2t^4 - 1\,dt = 2\left[\frac{2}{5}t^5 - t\right]_0^1 = 2\left[\frac{2}{5} - 1\right] = 2\left[-\frac{3}{5}\right] = -\frac{6}{5}$$

$$\neq 0$$

not orthogonal