MAT 202, Exam #1, Part I, Summer 2017 Name KEY

Instructions: Show all work. You may **not** use a calculator on this portion of the exam. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals). Reduce as much as possible. Be sure to complete all parts of each question. Provide explanations where requested. When you are finished with this portion of exam, get Part II.

1. Determine if each statement is True or False. (1 points each)

a. T **F** If a system of linear equations is consistent, then it has infinitely many solutions. only if also dependent

b. T **F** A linear system may have exactly two solutions. only 0, 1 or ∞ is possible

c. **T** F A homogeneous system of four linear equations in six variables has infinitely many solutions.

d. **T** F Multiplying a row of a matrix by a constant is one of the elementary row operations.

e. **T** F The matrix equation $A\vec{x} = \vec{b}$, where A is the coefficient matrix and $\vec{x}$ and $\vec{b}$ are column matrices, can be used to express a system of linear equations.

f. T **F** Matrix multiplication is commutative. it is not commutative (it is associative)

g. **T** F Every matrix A has an additive inverse.

h. T **F** If A can be row-reduced to the identity, then A is singular. it is nonsingular

i. T **F** The matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible when $ab - dc \neq 0$. $ad - bc \neq 0$

j. T **F** If A is a square matrix, then the system of linear equations $A\vec{x} = \vec{b}$ has a unique solution. it may have 1, 0, or ∞'ly many

k. T **F** If E is an elementary matrix, then $2E$ is an elementary matrix. multiple row operations

l. T **F** The zero matrix is an elementary matrix. not an allowable row operation

m. T **F** The product of a 2×3 matrix and a 3×5 matrix is a 5×2 matrix. 2×5 matrix

n. T **F** If A and B are nonsingular $n \times n$ matrices, then $A + B$ is a nonsingular matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

nonsingular nonsingular singular

2. Solve the system of equations $\begin{cases} 2x + 3y = -1 \\ x \ - y = \ 0 \end{cases}$ by writing the system as an augmented matrix and row-reducing by hand. (5 points)

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & 0 \end{bmatrix} \quad R_1 + (-1)R_2 \to R_1$$

$$\begin{bmatrix} 1 & 4 & -1 \\ 1 & -1 & 0 \end{bmatrix} \quad R_1 + (-1)R_2 \to R_2$$

$$\begin{bmatrix} 1 & 4 & -1 \\ 0 & 5 & -1 \end{bmatrix} \quad \tfrac{1}{5}R_2 \to R_2$$

$$\begin{bmatrix} 1 & 4 & -1 \\ 0 & 1 & -\tfrac{1}{5} \end{bmatrix} \quad -4R_2 + R_1 \to R_1$$

$$\begin{bmatrix} 1 & 0 & -\tfrac{1}{5} \\ 0 & 1 & -\tfrac{1}{5} \end{bmatrix}$$

$x = -\tfrac{1}{5}$

$y = -\tfrac{1}{5}$

3. Solve the same system in #2 but writing the system as a matrix equation and using inverse matrix methods. (5 points)

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2-3} \begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -\frac{1}{5} \begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 1 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} -1 + 0 \\ -1 + 0 \end{bmatrix} = \begin{bmatrix} -\tfrac{1}{5} \\ -\tfrac{1}{5} \end{bmatrix}$$

4. For the matrices $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}, D = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$. Calculate the following matrices. If the operation is not defined, explain why not. (4 points each)

a. $C^T + 3D$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 15 \\ -6 \end{bmatrix} = \begin{bmatrix} 16 \\ -5 \end{bmatrix}$$

b. AB

$$\begin{bmatrix} -2+6 & 6-15 \\ -4-2 & 12+5 \end{bmatrix} = \begin{bmatrix} 4 & -9 \\ -6 & 17 \end{bmatrix}$$

c. CD

$$\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \end{bmatrix} = 5-2=3$$

d. A^{-1}

$$\frac{1}{2+12}\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1/14 & 3/14 \\ -2/7 & 1/7 \end{bmatrix}$$

e. A^2

$$\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 4-12 & -6-3 \\ 8+4 & -12+1 \end{bmatrix} = \begin{bmatrix} -8 & -9 \\ 12 & -11 \end{bmatrix}$$

f. B^T

$$\begin{bmatrix} -1 & -2 \\ 3 & 5 \end{bmatrix}$$

g. $2A-3B$

$$\begin{bmatrix} 4 & -6 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -9 \\ 6 & -15 \end{bmatrix} = \begin{bmatrix} 7 & -15 \\ 14 & -13 \end{bmatrix}$$

5. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find a condition (or conditions) on a, b, c, d so that $AA^T = [\mathbf{0}]$. (6 points)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{bmatrix}$$

$$\boxed{\begin{matrix} a^2+b^2=0 \\ c^2+d^2=0 \end{matrix}} \qquad ac+bd=0$$

$$\Longrightarrow \quad a=b=c=d=0$$

6. Write the 3×3 elementary matrix E to perform the following row operations. (3 points each)

a. $2R_1 + R_2 \to R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b. $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

7. Find the LU factorization of $A = \begin{bmatrix} 1 & 7 \\ 2 & 20 \end{bmatrix}$. (5 points)

$$\begin{bmatrix} 1 & 7 \\ 2 & 20 \end{bmatrix} \quad -2R_1 + R_2 \to R_2 \qquad E_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 7 \\ 0 & 6 \end{bmatrix}$$

$$L = E_1^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 7 \\ 0 & 6 \end{bmatrix}$$

MAT 202, Exam #1, Part II, Summer 2017 Name ______________________

Instructions: Show all work. You **may** use a calculator on this portion of the exam. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question. Provide explanations where requested.

1. Solve the system $\begin{cases} 2x_1 + 2x_2 - x_3 \qquad = 1 \\ \qquad -x_2 \qquad + 3x_4 = 2 \end{cases}$ and write the solution in parametric form. (5 points)

$$\left[\begin{array}{cccc|c} 2 & 2 & -1 & 0 & 1 \\ 0 & -1 & 0 & 3 & 2 \end{array}\right] \Rightarrow \text{rref} \Rightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1/2 & 3 & 5/2 \\ 0 & 1 & 0 & -3 & -2 \end{array}\right]$$

$x_1 = 1/2x_3 - 3x_4 + 5/2$

$x_2 = 3x_4 - 2$

$x_3 = x_3$

$x_4 = x_4$

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix} t + \begin{bmatrix} -3 \\ 3 \\ 0 \\ 1 \end{bmatrix} s + \begin{bmatrix} 5/2 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$

2. Determine the value of a for which $\left[\begin{array}{cc|c} a & 1 & 1 \\ 2 & a-1 & 1 \end{array}\right]$ is consistent. (5 points)

$-\frac{2}{a}R_1 + R_2 \to R_2$

$-2 \quad -\frac{2}{a} \quad -\frac{2}{a}$

$$\left[\begin{array}{cc|c} a & 1 & 1 \\ 0 & a-1-\frac{2}{a} & 1-\frac{2}{a} \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} a & 1 & 1 \\ 0 & \frac{2}{a}-1 & 1-\frac{2}{a} \end{array}\right]$$

$\frac{2}{a} - 1 \neq 0$

$\frac{2}{a} \neq 1$

$a \neq 2$

if $a=2$ $\left[\begin{array}{cc|c} 2 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$

$a=0$ $\left[\begin{array}{cc|c} 0 & 1 & 1 \\ 0 & -1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 0 & 1 & 1 \\ 0 & 0 & 2 \end{array}\right]$

$\boxed{a \neq 0}$ for consistent system

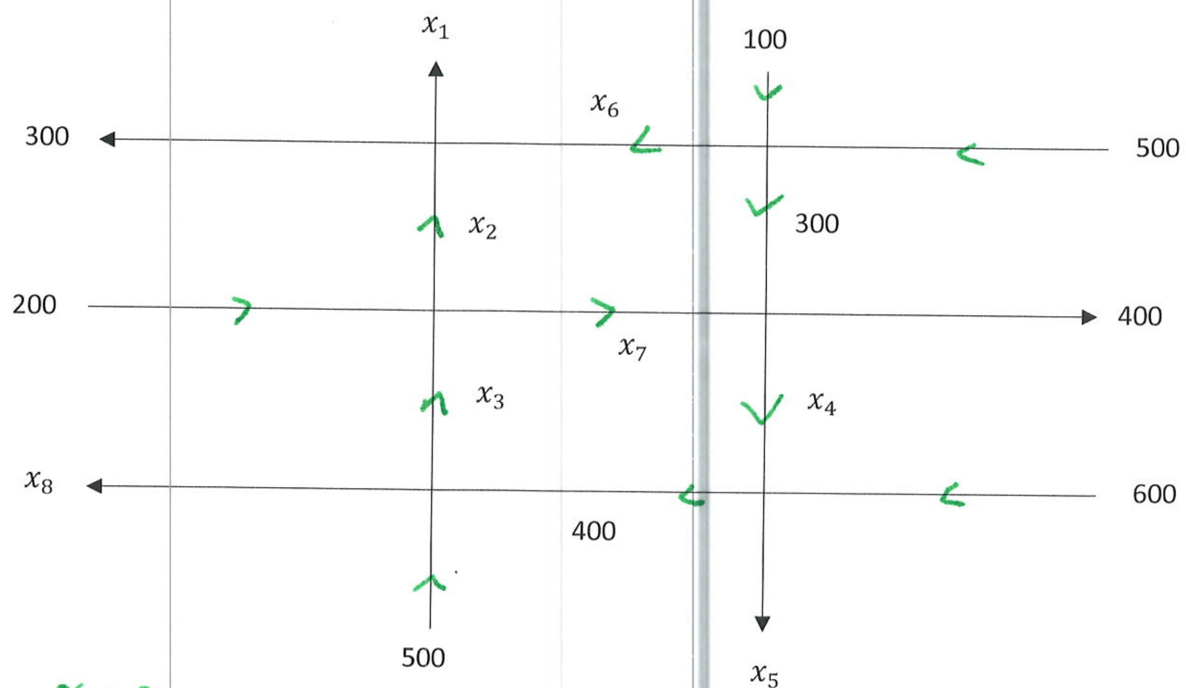
$a \neq 2$ for independence

3. Determine whether the solution to $\begin{cases} -x \quad\quad\; - z = 0 \\ -3x + y - 3z = 0 \\ \;\; x - 3y + 2z = 0 \end{cases}$ is trivial or non-trivial. Explain your reasoning. (5 points)

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ -3 & 1 & -3 & 0 \\ 1 & -3 & 2 & 0 \end{array}\right] \Rightarrow \text{rref} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

the solution is trivial since $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

4. Consider the traffic flow graph below. Write the matrix to the system and solve it. If the solution is dependent, write it in parametric form. (7 points)



$x_2 + x_6 = x_1 + 300 \rightarrow -x_1 + x_2 + x_6 = 300$

$x_3 + 200 = x_2 + x_7 \qquad x_2 - x_3 + x_7 = 200$

$x_3 + x_8 = 400 + 500 \qquad x_3 + x_8 = 900$

$100 + 500 = x_6 + 300 \qquad x_6 = 300$

$300 + x_7 = 400 + x_4 \qquad -x_4 + x_7 = 100$

$x_4 + 600 = 400 + x_5 \qquad x_4 - x_5 = -200$

next page

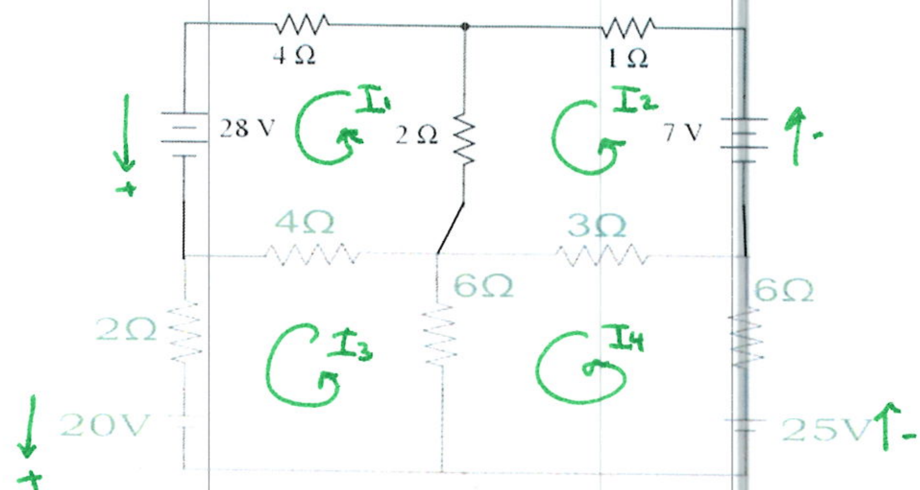
$$\left[\begin{array}{cccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1100 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1100 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 900 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & -100 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 300 \end{array}\right]$$

$$\begin{aligned} x_1 &= -x_7 - x_8 + 1100 \\ x_2 &= -x_7 - x_8 + 1100 \\ x_3 &= -x_8 + 900 \\ x_4 &= x_7 - 100 \\ x_5 &= x_7 + 100 \\ x_6 &= 300 \end{aligned}$$

$$x_7 = x_7$$
$$x_8 = x_8$$

$$\vec{x} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_7 + \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_8 + \begin{bmatrix} 1100 \\ 1100 \\ 900 \\ -100 \\ 100 \\ 300 \\ 0 \\ 0 \end{bmatrix}$$

5. Set up and solve the loop circuit diagram below. Round your values for the currents to three significant digits. (7 points)



$$10I_1 - 2I_2 - 4I_3 = 28$$
$$-2I_1 + 6I_2 - 3I_4 = -7$$
$$-4I_1 + 12I_3 - 6I_4 = 20$$
$$-3I_2 - 6I_3 + 15I_4 = -25$$

$$\left[\begin{array}{cccc|c} 10 & -2 & -4 & 0 & 28 \\ -2 & 6 & 0 & -3 & -7 \\ -4 & 0 & 12 & -6 & 20 \\ 0 & -3 & -6 & 15 & -25 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3.79 \\ 0 & 1 & 0 & 0 & -.244 \\ 0 & 0 & 1 & 0 & 2.59 \\ 0 & 0 & 0 & 1 & -.68 \end{array}\right]$$

6. Encode the message NO MAN CAN SERVE TWO MASTERS with the matrix $A = \begin{bmatrix} 6 & 8 & 1 & -5 \\ 6 & -9 & 7 & 6 \\ -4 & 6 & 1 & -9 \\ -1 & 1 & 1 & 3 \end{bmatrix}$ using 0 as a space and the corresponding position (number) in the alphabet for letters. (6 points)

$$\begin{bmatrix} 14 & 15 & 0 & 13 \\ 1 & 14 & 0 & 3 \\ 1 & 14 & 0 & 19 \\ 5 & 18 & 22 & 5 \\ 0 & 20 & 23 & 15 \\ 0 & 13 & 1 & 19 \\ 20 & 5 & 18 & 19 \end{bmatrix}$$

$\Rightarrow$
161 -10 132 59 87 -115 102 88
71 -99 118 136 45 15 158 -100
13 -27 178 -42 55 -92 111 126
59 242 92 -175

7. Use a system of equations to write the partial fraction decomposition of the rational expression below. Use the matrix to solve the system. (6 points)

$$\frac{4x^2}{(x+1)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$A(x^2+2x+1) + B(x^2-1) + C(x-1) = 4x^2$

$Ax^2 + 2Ax + A + Bx^2 - B + Cx - C = 4x^2$

$Ax^2 + Bx^2 = 4x^2 \Rightarrow A + B = 4$

$2Ax + Cx = 0 \Rightarrow 2A + C = 0$

$A - B - C = 0 \Rightarrow A - B - C = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 2 & 0 & 1 & 0 \\ 1 & -1 & -1 & 0 \end{array}\right] \Rightarrow \text{rref}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$A = 1$

$B = 3$

$C = -2$

$$\frac{4x^2}{(x+1)^2(x-1)} = \frac{1}{x-1} + \frac{3}{x+1} - \frac{2}{(x+1)^2}$$