MAT 202, Quiz #7, Summer 2017 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. For $\vec{u} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \vec{v} = \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}, \vec{w} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, find:
   a. $\vec{u} + \vec{v}$

   $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

   b. $\vec{u} - 2\vec{v} + 3\vec{w}$

   $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ -8 \\ 0 \end{bmatrix} + \begin{bmatrix} 6 \\ 3 \\ -3 \end{bmatrix} = \begin{bmatrix} 11 \\ -7 \\ 0 \end{bmatrix}$

2. The properties a vector space must obey are listed below. Argue for/prove that the set of all polynomials of degree at most n is a vector space. Be sure to address each of the properties in some way.

1. $\mathbf{u} + \mathbf{v}$ is in V.
2. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
3. $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$
4. V has a **zero vector** $\mathbf{0}$ such that for every $\mathbf{u}$ in V, $\mathbf{u} + \mathbf{0} = \mathbf{u}$.
5. For every $\mathbf{u}$ in V, there is a vector in V denoted by $-\mathbf{u}$ such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$.
6. $c\mathbf{u}$ is in V.
7. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
8. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
9. $c(d\mathbf{u}) = (cd)\mathbf{u}$
10. $1(\mathbf{u}) = \mathbf{u}$

$p(x), q(x), r(x)$ in P_n, ie.: $a_n x^n + a_{n-1}x^{n-1} + \dots + a_0$

$p+q$ is still a poly of degree n or less, no change in degree each

$p+q = q+p$ since we are adding real coefficients

$p + (q+r) = (p+q) + r$ since we are adding real coeff.

$\vec{0}$ in $P_n = 0$ (all coeff are zero)

$p(x) + 0 = p(x)$

$-p(x) + p(x) = 0$ since we are adding real coeff.

$cp(x)$ in P_n since scaling does not change degree of $p(x)$

$c(p+q) = cp(x) + cq(x)$ adding real coeff.

$(c+d)p = cp(x) + dp(x)$ by prop of real coeff.

$(cd)p(x) = c(d\,p(x))$ by real coeff.

$1p(x) = p(x)$