Chapter 2: Things To Know

Section 2.1: Perimeter, Circumference, and Area

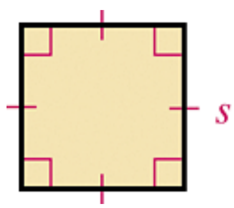
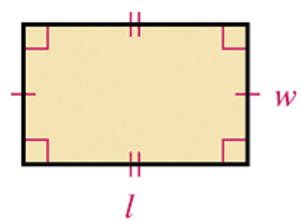
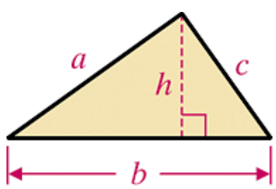
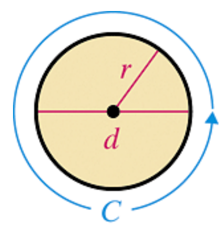
Objectives:	Vocabulary:
1. Find the Perimeter of Circumference of Basic Shapes 2. Find the Area of Basic Shapes	• perimeter • circumference • area

The ___________________ , P, of a geometric figure is the distance around the figure.

The distance around a circle is given a special name, called its __________________________ , C.

The ________________ A of a geometric figure is the number of square units it encloses.

Basic Perimeter and Area formulas:

<table>
<tr>
<td>Square
side length:
$P =$
$A =$

</td>
<td>Rectangle
length: and width:
$P =$
$A =$

</td>
</tr>
<tr>
<td>Triangle
side lengths:
base: and height:
$P =$
$A =$

</td>
<td>Circle
radius: and diameter:
$C =$ or $C =$
$A =$

</td>
</tr>
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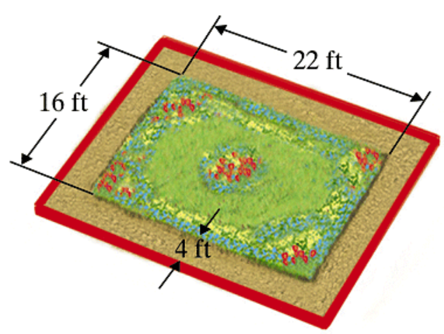
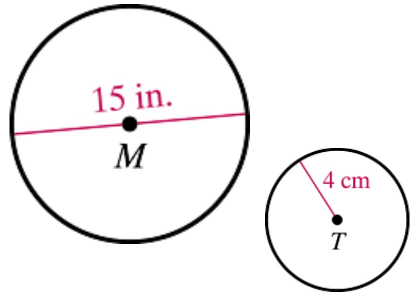
(We will come back to these formulas later in the semester.)

Example Finding the Perimeter of a Rectangle

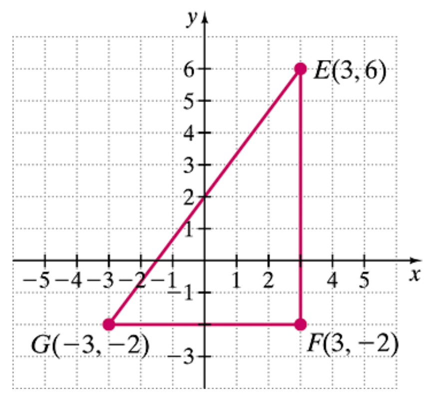
The botany club members are designing a rectangular garden for the courtyard of your school as shown to the right. They plan to place edging on the outside of the path. How much edging material will they need?



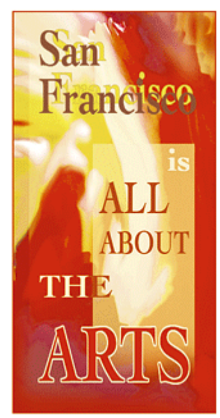
Example Finding Circumference
What is the circumference of each circle in terms of pi? What is the circumference of the circle to the nearest tenth?



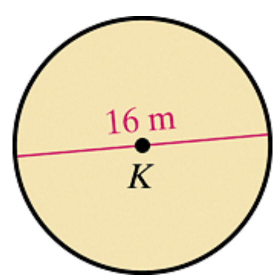
Example Finding Perimeter in the Coordinate Plane
What is the perimeter of ΔEFG?



Example Finding the Area of a Rectangle
We want to make a rectangular banner similar to the one at the right. The banner shown is 2 ½ ft in width and 5 ft in length. If we round up to the nearest square yard, how much material do we need?



Example Finding the Area of a Circle
Find the area of the circle in terms of pi.



Section 2.2: Patterns and Inductive Reasoning

Objectives:	Vocabulary:
1. Use Logic to Understand Patterns. 2. Understand and Use Inductive Reasoning. 3. Form Conjectures and Find Counterexamples.	• inductive reasoning • induction • conjecture • counterexample

Example Using Logic
Look for a pattern in each list. Then use this pattern to predict the next number.

a. 3, 4, 6, 9, 13, 18, …

b. 3, 6, 18, 36, 108, 216, …

_________________________ is the process of arriving at a general conclusion based on observing patterns or observing specific examples.

These conclusions are called ________________ or ________________________.

Example Using Inductive Reasoning
Look for a pattern in each list. Then use this pattern to predict the next number.

a. 1, 1, 2, 3, 5, 8, 13, 21, …

b. 23, 54, 95, 146, 117, 98…

Example Using Inductive Reasoning
Notice two patterns in this sequence of figures. Use the patterns to draw the next figure in the sequence.

Example Forming Conjectures
Study the list of circles. Use the pattern to answer the questions.

a. Make a conjecture about the color of the 11^{th} circle.

b. Make a conjecture about the number of regions in the 11^{th} circle.

c. Make a conjecture about the appearance of the 11^{th} circle.

d. Make a conjecture about the appearance of the 30^{th} circle.

A ______________________________ is an example that shows a conjecture is false or incorrect.

Example Finding a Counterexample
Find a counterexample to show that each conjecture is false.

a. **Conjecture:** The product of two numbers is always greater than either number.

Can you think of any other counterexamples than the one provided in the PowerPoint?

b. **Conjecture:** All apples are red.

Section 2.3: Conditional Statements

Objectives:	Vocabulary:
1. Recognize Conditional Statements and Their Parts. 2. Write Converses, Inverses, and Contrapositives of Conditional Statements.	• conditional statement • "if-then" form • hypothesis • conclusion • negation • converse • inverse • contrapositive • equivalent statements

A ______________________________________ is a statement that is written, or that can be written, in ____________________ form.

When a statement is in "if-then" form, the phrase that follows "if" is called the __________________, and the phrase that follows "then" is the ____________________.

Make up your own example of a conditional statement below:

Symbols Shorthand for "if-then" notation		
Hypothesis		**Conclusion**
if p		q
p		q
p		q

Example Identifying the Hypothesis and the Conclusion
Identify the hypothesis (p) and the conclusion (q).

a. If an animal is a turtle, then the animal is a reptile.

b. If a number is even, then the number is not odd.

Example Writing a Conditional Statement
A hypothesis (p) and a conclusion (q) are given. Use them to write a conditional statement, $p \rightarrow q$.

a. p: a figure is a square

   q: the figure is not a triangle

b. p: 9 is a perfect square

   q: 9 is not a prime number.

Example Writing a Conditional Statement
Write the following statements in "if-then" form.

Acute angles measure less than 90°.

A conditional statement may be either *true* or *false*.

- A conditional statement is true if ____

  ____.

- A conditional statement is false if ____

  ____.

Example Is a Conditional Statement True or False?
Determine whether each conditional statement is true or false.

a. If an angle measures 92°, then it is an obtuse angle.

b. If a month begins with the letter J, then the month has 31 days.

The ______________________ of a statement is formed by writing the negative of the statement.

> Note: There's a typo in this slide of the PowerPoint. The symbol for "not *p*" is $\sim p$.

> Make up your own example to illustrate negating a statement below.

Read the table in the PowerPoint carefully—especially the example. Below, fill in just the names and symbols column from the table.

Related Conditional Statements	
Name	**Symbols**

Example Writing Related Conditional Statements
Write the (a) converse, (b) inverse, and (c) contrapositive of the given conditional statement.

If it is raining, then it is cloudy.

Section 2.4: Biconditional Statements and Definitions

Objectives:	Vocabulary:
1. Write and Understand Biconditional Statements. 2. Identify and Understand Good Definitions	• biconditional statement • good definition

A ____ is equivalent to writing a conditional statement and its converse. For a biconditional statement, we use the phrase __________________________________.

Example Rewriting a Biconditional Statement
Write the given biconditional statement as a conditional statement and its converse.

Two angles are supplementary if and only if the sum of their measures is 180°.

Notation

"p if and only if q" can be written __________________

Example Writing a True Biconditional Statement
In each of the following examples, a true conditional statement is given.

a. Write the converse of the conditional statement.
b. Decide whether the converse statement is true or false.
c. If the converse statement is true, write a true biconditional statement. If the converse statement is false, give a counterexample.

I. If a closed figure is a triangle, then it has three sides.

II. If $x = 5$, then $x^2 = 25$.

Example Identifying Good Definitions

Multiple Choice Which of the following is a good definition?

a. A fish is an animal that swims.

b. Rectangles have four corners.

c. Giraffes are animals with very long necks.

d. A penny is a coin worth one cent.

Example Writing a Definition as a True Biconditional

Determine whether or not the following definition is a good one. To do so, attempt to write it as a true biconditional statement.

Definition: A right angle is an angle that measures 90°.

Section 2.5: Deductive Reasoning

Objectives:	Vocabulary:
• Review Conditional, Converse, Inverse, and Contrapositive Statements. • Understand and Use Two Laws of Deductive Reasoning: the Law of Detachment and the Law of Syllogism.	• deductive reasoning • Law of Detachment • Law of Syllogism

To begin, let's review related conditional statements. (Writing these things over again can't hurt!)

Related Conditional Statements			
Name	***How to write it (words)***	***How to write it (symbols)***	***How to say it (words)***

Example Writing Related Conditional Statements
Use the given conditional statement to write its (a) converse, (b) inverse, and (c) contrapositive statements in words and in symbols.

Conditional: If a number is a whole number, then it is an integer. ($p \rightarrow q$)

Converse:

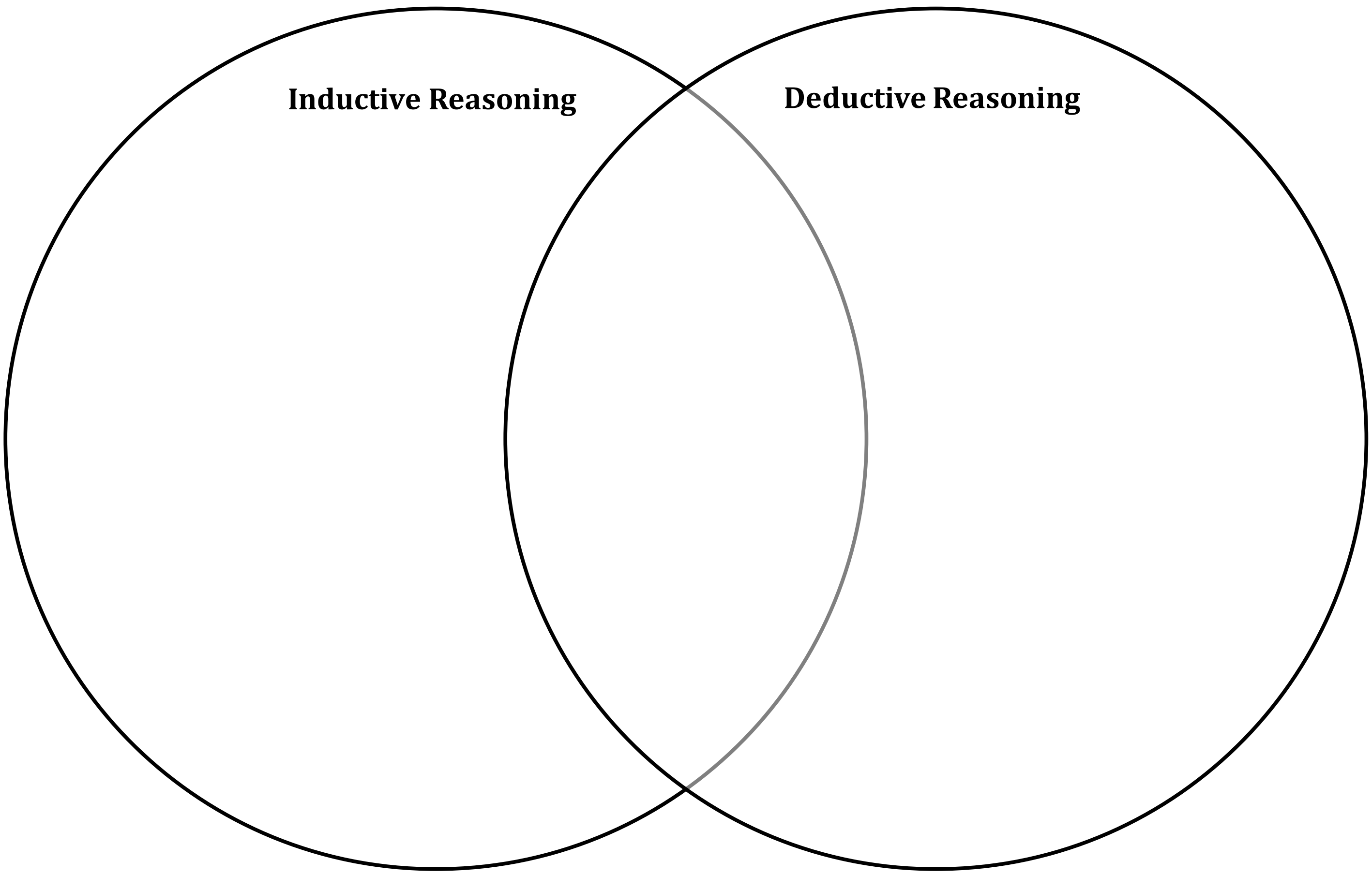
Inverse:

Contrapositive:

____ is the process of proving a specific conclusion from one or more general statements. A conclusion that is proved true by deductive reasoning is called a theorem.

Inductive vs. Deductive Reasoning

Fill in the venn diagram below to compare and contrast inductive and deductive reasoning.



Law of Detachment *(Write the Law of Detachment in the space below.)*

Example Using the Law of Detachment

Use the Law of Detachment to make a true conclusion. Assume that the first statement ($p \rightarrow q$) is true.

If you shop at Save Market, you will save money.

You shop at Save Market.

Conclusion:

Example Using the Law of Detachment
Determine whether each reasoning is valid using the Law of Detachment. Assume that the first statement $(p \rightarrow q)$ is true.

a. If any student gets an A on a final exam, then the student will pass the course.
   Leo got an A on his final exam, so he will pass the course.

b. If two angles are adjacent, then they share a common vertex.
   $\angle 1$ and $\angle 2$ share a common vertex, so the angles are adjacent.

Law of Syllogism *(Write the Law of Syllogism in the space below.)*

Example Using the Law of Syllogism
Use the Law of Syllogism to form a true conclusion $(p \rightarrow r)$.

Given: If a figure is a square, then the figure is a rectangle.
If a figure is a rectangle, then the figure has four sides.

Example Using the Laws of Deductive Reasoning
Decide what you can conclude from the true conditional statements given, and note whether your reasoning involves the Law of Detachment or the Law of Syllogism.

a. **Given:** All elephants are mammals.
   All mammals have hair.

b. **Given:** If a number is prime, then it has exactly two factors.
   The number 11 is prime.

Section 2.6: Reviewing Properties of Equality and Writing Two-Column Proofs

Objectives:	Vocabulary:
1. Use Properties of Equality to Justify Reasons for Steps. 2. Write a Two-Column Proof.	• Reflexive Property • Symmetric Property • Transitive Property • Substitution Property • proof • two-column proof

Review Algebra Properties of Equality Complete the following table.

Let a, b, and c be any real numbers.		
Addition Property	If…	then…
Subtraction Property	If…	then…
Multiplication Property	If…	then…
Division Property	If…	then…
Reflexive Property		
Symmetric Property	If…	then…
Transitive Property	If…	then…
Substitution Property	If…	then…

Note: Depending on your computer's version of PowerPoint, the multiplication symbol × may show up as . Can you figure out why?

The Distributive Property *(Write the distributive property in the space below. Just the algebra is fine.)*

Example Giving Reasons for Statements

Solve $5x + 12 = 47$. Give a reason to justify each statement.

Statements	Reasons

Example Giving Reasons for Statements

Solve $7x - 10(5 + 3x) = 2x$. Give a reason to justify each statement.

Statements	Reasons

Geometric Properties of Equality Complete the following table.

Properties of Equality		
	Angle Measures	***Segment Lengths***
Reflexive Property		
Symmetric Property		
Transitive Property		

Example Stating Properties

Fill in each blank with the reason to justify the statement.

(**Given** statements are the "if" part. You are filling in the reasons for the "then" part after each "?".)

Statements	Reasons
a. $CG = MN$ $MN = CG$	Given ?
b. $m\angle P = m\angle Q$ $m\angle Q = m\angle B$ $m\angle P = m\angle B$	Given Given ?
c. $m\angle 1 + m\angle 2 + m\angle 3 = 50°$ $m\angle 1 + m\angle 2 = 43°$ $43° + m\angle 3 = 50°$	Given Given ?

Writing a Two-Column Proof

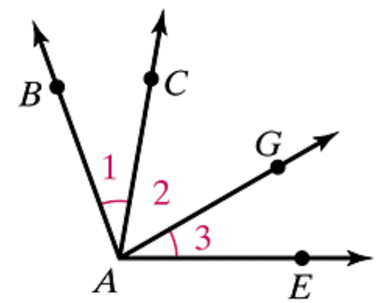
A ___________________ is an argument that uses logic to establish the truth of a statement.

A ____ lists numbered statements on the left and corresponding numbered reasons or justifications on the right. These statements show the logical order of the proof.

> We will get a **LOT** more practice writing proofs during class time, but for now, make sure you can follow the chain of logic in the proof below all the way from the "Given" statement to the desired conclusion.

Example Writing a Two-Column Proof
Write a two-column proof.

Given: $m\angle 1 = m\angle 3$
Prove: $m\angle BAG = m\angle EAC$



Proof strategy: *Use this space to write about the logical argument you are going to make.*

Statements	Reasons
1. $m\angle 1 = m\angle 3$	1. Given

Section 2.7: Proving Theorems About Angles

Objectives:	Vocabulary:
• Prove and Use Theorems about Angles.	No new vocabulary

When writing a proof, you will always want to keep two questions in mind:
(1) What do you know?, and
(2) What do you want to know?

Things that you know may be information given to you, or these facts may arise from using postulates or theorems.

In general, you will be able to find LOTS of information about a given figure—not all of it will be relevant to your proof! This is why you want to keep the question "What do I want to know?" in mind.

It may also be useful to write the "Given" and "Prove" statements into "if-then" form. In this form,

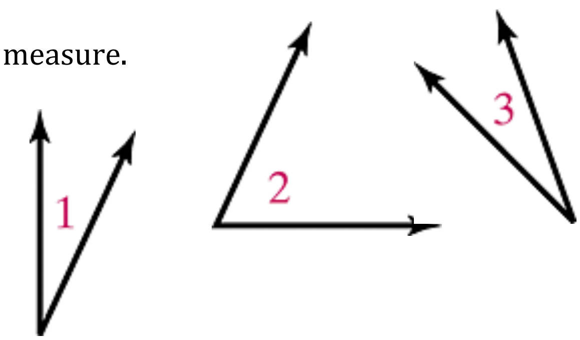
Given is the ____________________________ and follows the word ______________.

Prove is the ____________________________ and follows the word ______________.

Theorem Equal Complements Theorem

Complements of the same angle (or of equal angles) are equal in measure.

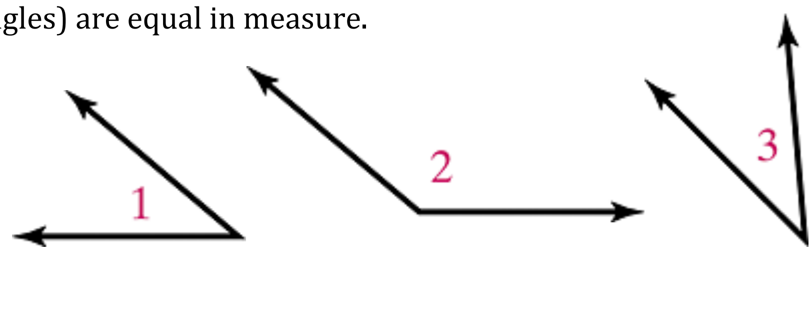
Write the "if-then" statement of this theorem.



Theorem Equal Supplements Theorem

Complements of the same angle (or of equal angles) are equal in measure.

Write the "if-then" statement of this theorem.



We will prove the Equal Complements Theorem (The proof of the Equal Supplements Theorem looks almost identical to it.)

Example Proving the Equal Complements Theorem

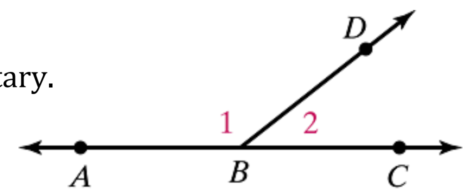
Given: $\angle A$ and $\angle B$ are complementary.
$\angle C$ and $\angle D$ are complementary.
$m\angle A = m\angle C$

Prove: $m\angle B = m\angle D$

Statements	Reasons

Theorem Linear Pair Theorem

If two angles form a linear pair, then the angles are supplementary.



Example Proving the Linear Pair Theorem

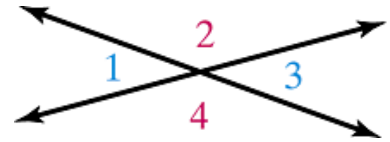
Given: $\angle 1$ and $\angle 2$ form a linear pair.

Prove: $\angle 1$ and $\angle 2$ are supplementary angles.

Statements	Reasons

Theorem Vertical Angles Theorem

Vertical angles are congruent.



You may be thinking about now, "Do I *really* need to remember all of the names of these theorems that say obvious things?" The answer is no, you don't need to remember the names of these theorems, but if, 3 weeks from now, you look at the above picture and make the claim that $\angle 2 \cong \angle 4$, I will ask how you know that's true, and you will say "Because vertical angles are congruent!"

Example Proving the Vertical Angles Theorem

Given: $\angle 1$ and $\angle 3$ are vertical angles. **Prove:** $\angle 1 \cong \angle 3$

Statements	Reasons

Theorem Right Angles Congruent Theorem

All right angles are congruent.

Theorem Equal Supplementary Angles Theorem

Two equal supplementary angles are right angles.

It's important to know that proofs can come in other forms than tables, but all the proofs we'll do this semester will be in table form. Don't worry too much about the paragraph proof—just read through it.

Example Using the Vertical Angles Theorem

Find the value of x.