

Chapter 4: Things To Know

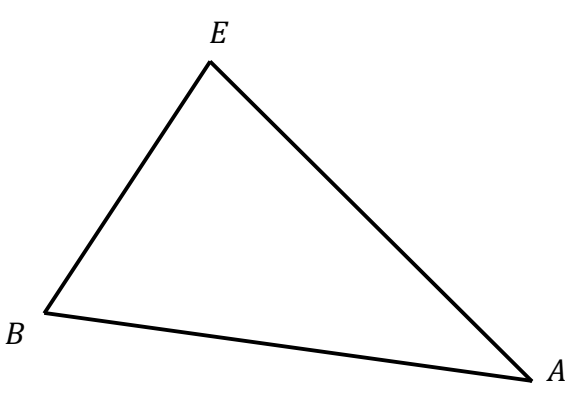
Section 4.1 Types of Triangles

<p>Objectives</p> <ol style="list-style-type: none"> 1. Learn the Vocabulary of Triangles. 2. Classify Triangles by Angles and Sides. 3. Find Angle Measures of Triangles. 	<p>Vocabulary</p> <ul style="list-style-type: none"> • triangle • vertex • sides of a triangle • adjacent sides • opposite side and angle • included side and angle • acute triangle • obtuse triangle • equiangular triangle • right triangle • scalene triangle • isosceles triangle • equilateral triangle • interior angle • exterior angle • corollary
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A _____ is formed by 3 noncollinear points connected by line segments.

The noncollinear points are called _____.

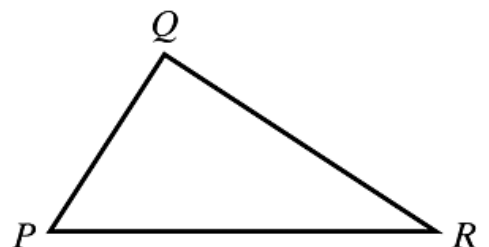
The segments joining the points are called _____.

<i>How to name it</i>	
<p>This shape is _____.</p> <p>Points A, B, and E are _____.</p> <p>Point A is a _____.</p> <p>Point B is a _____.</p> <p>Point E is a _____.</p> <p>\overline{AB}, \overline{BE}, and \overline{AE} are the _____.</p>	

Example Identifying Parts of a Triangle

Given $\triangle PQR$:

- a. Which angle is opposite \overline{PQ} ? _____
- b. Which side is opposite $\angle Q$? _____
- c. Which side is included between $\angle P$ and $\angle R$? _____
- d. Which angle is included between \overline{QR} and \overline{PR} ? _____



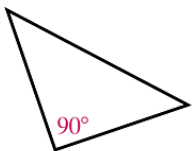
<i>Classifying Triangles by Angles</i>		
Type	Description	Example

<i>Classifying Triangles by Sides</i>		
Type	Description	Example

Example Classifying Triangles

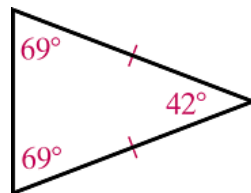
Classify each triangle by its angles and sides. Use the most specific name.

a.



No sides are equal

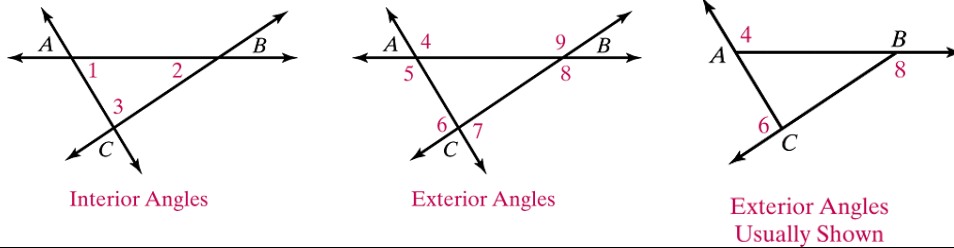
b.



Definitions

We call the 3 original angles of the triangle the _____ angles of the triangle.

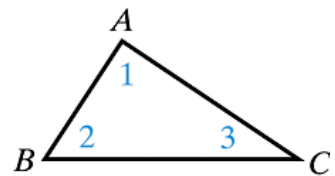
The angles that are adjacent to the interior angles are the _____ angles of the triangle. There are two exterior angles associated with each interior angle of a triangle, but since they are congruent to each other, we only show one exterior angle for each interior angle.



Triangle Angle-Sum Theorem

The sum of the measures of the interior angles of a triangle is _____.

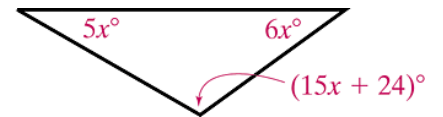
$m\angle 1 + m\angle 2 + m\angle 3 = \underline{\hspace{2cm}}$



Corollaries (Follow directly from the previous theorem)	
<p>Exterior Angle of a Triangle</p> <p>Write the equation here:</p>	<p>Acute Angles of a Right Triangle</p> <p>Write the equation here:</p>

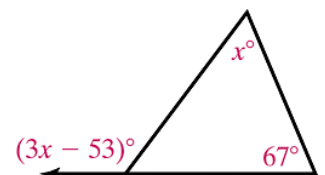
Example Finding Angle Measures

Use the Triangle Angle-Sum Theorem to find the measure of each angle in the given triangle.



Example Finding Angle Measures

Use the Exterior Angle of a Triangle Corollary to find the measure of the exterior angle and the nonadjacent angle shown.

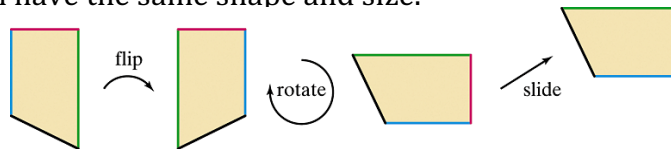


Section 4.2 Congruent Figures

Objectives	Vocabulary
<ol style="list-style-type: none"> 1. Identify Corresponding Parts in Congruent Figures. 2. Prove Triangles are Congruent. 	<ul style="list-style-type: none"> • congruent figures • corresponding sides • corresponding angles

_____ have the exact same shape and size.

A _____, a _____, or a _____ does not affect whether figures are congruent because they still have the same shape and size.

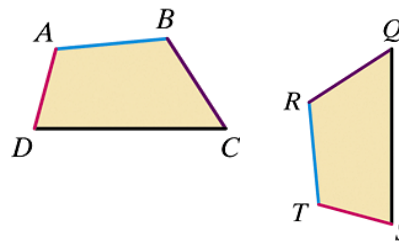


When figures are congruent, their _____ are congruent, and their _____ are congruent.

<p>The diagram shows two triangles, $\triangle ABC$ and $\triangle TRS$, which are congruent. Corresponding angles are marked with single arcs (at A and T, B and S, C and R). Corresponding sides are marked with double and triple tick marks. Callouts indicate 'congruent angles' and 'congruent sides'.</p>	<p>Corresponding Angles</p>	<p>Corresponding Sides</p>

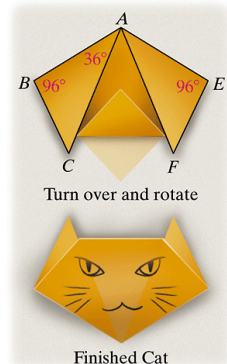
Example Naming Congruent Parts

For the two figures, we are given that $ABCD \cong TRQS$



Example Using Congruent Triangles to find Angle Measures

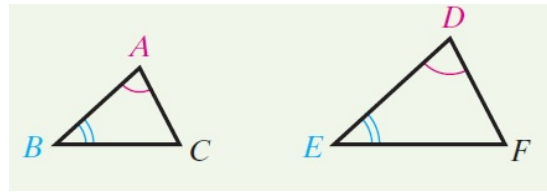
In the process of folding an origami cat, two congruent triangles are formed. Given that $\triangle ABC \cong \triangle AEF$, find $m\angle F$.



Theorem Third Angles Theorem

If ...

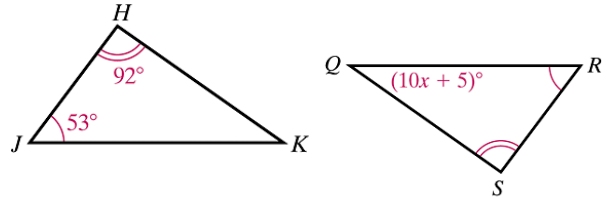
then...



In my humble opinion, this is one of the more ridiculous facts to be given the title of "Theorem" in this book. Don't focus on memorizing the **name** of it—just keep in mind that if two triangles have two angles congruent, **of course** the third angle has to be congruent, because both sets of angles have to add up to 180° .

Example Using the Third Angles Theorem

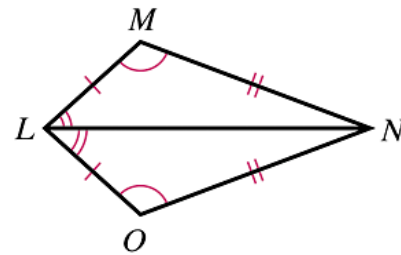
Find the value of x .



Example Proving Triangles are Congruent

Given: $\overline{LM} \cong \overline{LO}$, $\overline{MN} \cong \overline{ON}$, $\angle M \cong \angle O$, $\angle MLN \cong \angle OLN$

Prove: $\triangle LMN \cong \triangle LON$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

Section 4.3 Congruent Triangles by SSS and SAS

Objectives

1. Prove Two Triangles are Congruent Using the SSS and SAS Postulates.

Vocabulary

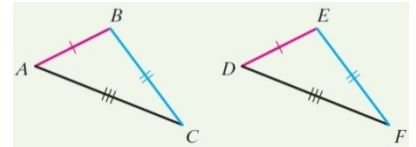
- Side-Side-Side (SSS)
- Side-Angle-Side (SAS)

In this section and the next, you'll learn some postulates and theorems that will be super useful for the types of proofs you did at the end of Section 4.2—Triangle Congruence Proofs!

Side-Side-Side (SSS) Postulate

If...

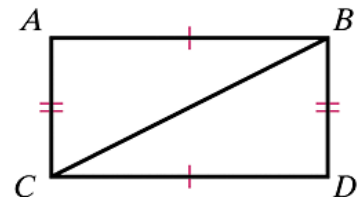
Then...



Example Proving Triangles are Congruent

Given: $\overline{AB} \cong \overline{CD}, \overline{AC} \cong \overline{BD}$

Prove: $\triangle ABC \cong \triangle DCB$

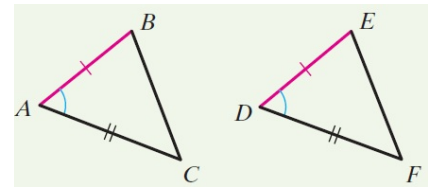


Statements	Reasons
1.	1.
2.	2.
3.	3.

Side-Angle-Side (SAS) Postulate

If...

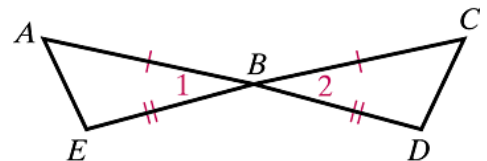
Then...



Example Proving Triangles are Congruent

Given: The figure with congruent segments marked.

Prove: $\triangle ABE \cong \triangle CBD$

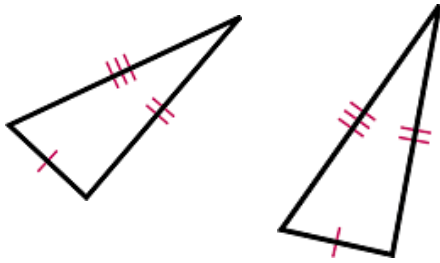


Statements	Reasons
1.	1.
2.	2.
3.	3.

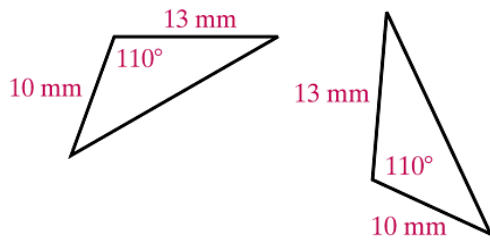
Example Identifying Congruent Triangles

Can we use SSS or SAS to prove the triangles are congruent? If there is not enough information to prove by SSS or SAS, then write “not enough information” and explain why.

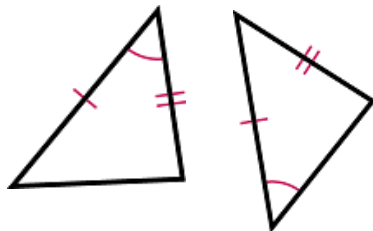
a.



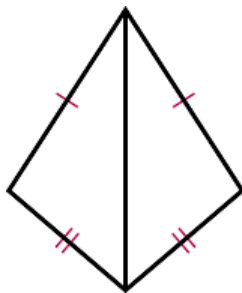
b.



c.



d.



Section 4.4 Congruent Triangles by ASA and AAS

Objectives

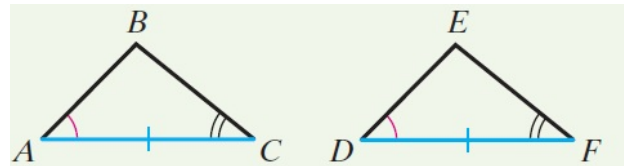
1. Prove Two Triangles Are Congruent Using the ASA Postulate and the AAS Theorem.
2. Identify When to Use SSS, SAS, ASA, or AAS to Prove Triangles Congruent.

Vocabulary

- Angle-Side-Angle (ASA)
- Angle-Angle-Side (AAS)

Angle-Side-Angle (ASA) Postulate

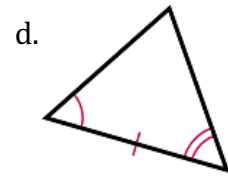
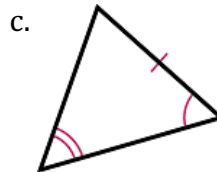
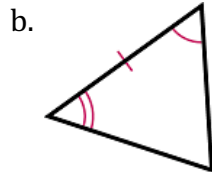
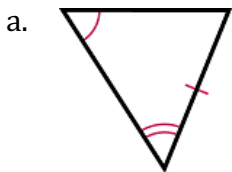
If...



Then...

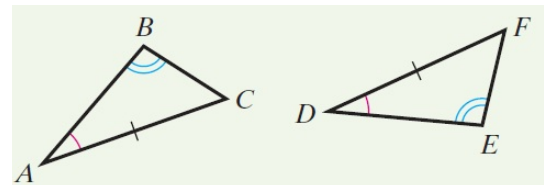
Example Identifying ASA

Multiple Choice: Choose two triangles that are congruent by the ASA Postulate. Explain why.



Angle-Angle-Side (AAS) Theorem

If...



Then...

Why is AAS called a Theorem and not a Postulate?

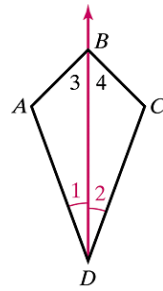
Because we can prove that it's true using the ASA Postulate:

Since $\angle A \cong \angle D$ and $\angle B \cong \angle E$, that means that $\angle C \cong \angle F$. (By the Third Angle Theorem!)

Example Proving Triangles are Congruent

Given: \overline{DB} bisects $\angle ABC$, $\angle 1 \cong \angle 2$

Prove: $\triangle DAB \cong \triangle DCB$

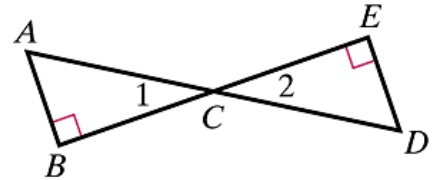


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

Example Proving Triangles are Congruent

Given: $\angle B$ and $\angle E$ are right angles, C is the midpoint of \overline{AD} .

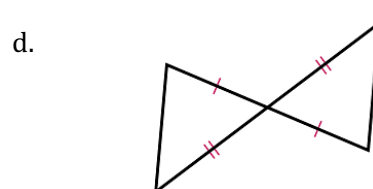
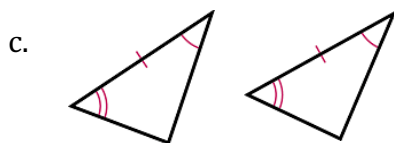
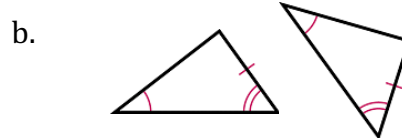
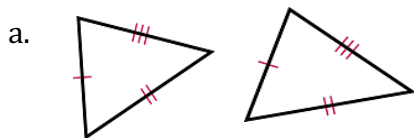
Prove: $\triangle ABC \cong \triangle DEC$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Example Identifying SSS, SAS, ASA, and AAS

Each pair of triangles is congruent by SSS, SAS, ASA, or AAS. Identify the postulate or theorem that confirms their congruence.



Section 4.5 Proofs Using Congruent Triangles

Objectives

1. Use Triangle Congruence and Corresponding Parts of Congruent Triangles to Prove that Parts of Two Triangles Are Congruent.
2. Prove Two Triangles are Congruent Using Other Congruent Triangles.

Vocabulary

- cpoctac (Corresponding Parts Of Congruent Triangles Are Congruent)

Often we are interested in proving that two lengths, or angles, or other types of measures are the same—i.e. that two geometric figures are congruent. If we can prove that two **triangles** are congruent, then we can prove that **corresponding parts of those congruent triangles are congruent**. We abbreviate this with the following acronym.

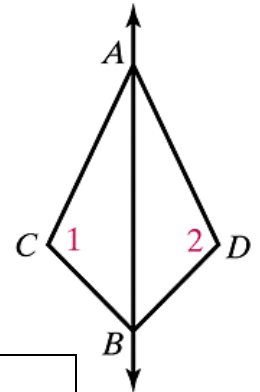
cpoctac = Corresponding Parts Of Congruent Triangles Are Congruent

Example Using Congruent Triangles

Given: \overline{AB} bisects $\angle CAD$

\overline{BA} bisects $\angle CBD$

Prove: $\angle 1 \cong \angle 2$

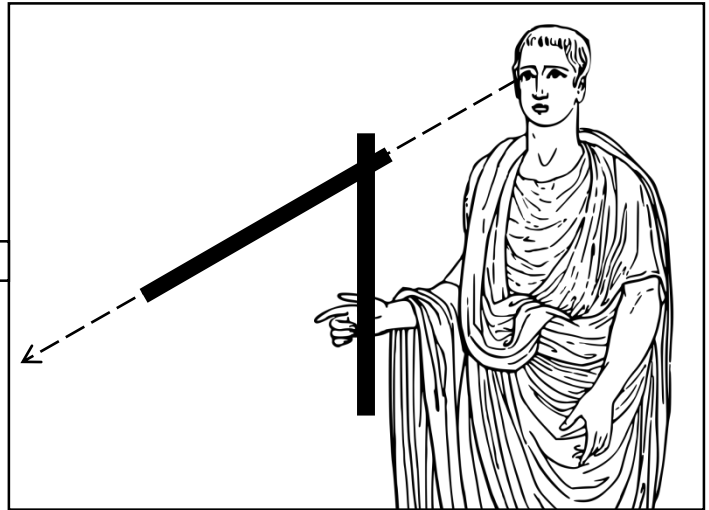
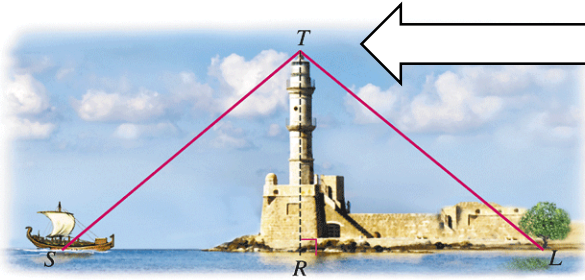


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Example Proving Triangle Parts Congruent to Measure Distance

Thales, a Greek philosopher, is said to have developed a method to measure the distance to a ship at sea.

He made a compass by nailing two sticks together. Standing on top of a tower, T , he would hold one stick vertical and tilt the other until he could see the ship S along the line of the tilted stick.



With this compass setting, he would find a landmark L on the shore along the line of the tilted stick. How far would the ship be from the base of the tower?

Given: $\angle TRS$ and $\angle TRL$ are right angles.

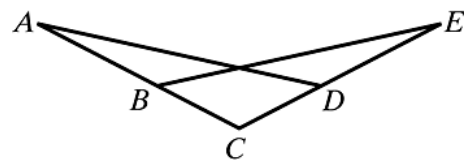
$$\angle RTS \cong \angle RTL$$

Prove: $\overline{RS} \cong \overline{RL}$

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Example Identifying Common Parts of Overlapping Triangles

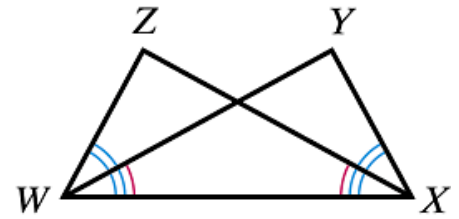
What common angle do $\triangle ACD$ and $\triangle ECB$ share?



Example Using Common Parts of Overlapping Triangles

Given: $\angle ZXW \cong \angle YWX, \angle ZWX \cong \angle YXW$

Prove: $\overline{ZW} \cong \overline{YX}$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

Section 4.6 Isosceles, Equilateral, and Right Triangles

Objectives

1. Use Properties of Isosceles and Equilateral Triangles.
2. Use Properties of Right Triangles.

Vocabulary

- legs of an isosceles triangle
- base of an isosceles triangle
- vertex angle of an isosceles triangle
- base angles of an isosceles triangle
- hypotenuse
- legs of a right triangle

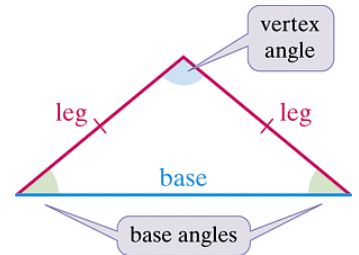
Isosceles Triangles

If an isosceles triangle has exactly two congruent sides, then these two sides are its _____.

The third side is the _____.

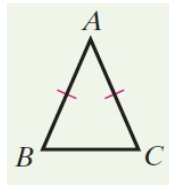
The angle opposite the base is the _____.

The other two angles adjacent to the base are the _____.

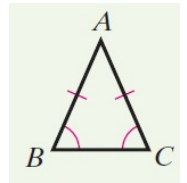


Isosceles Base Angles Theorem

If...

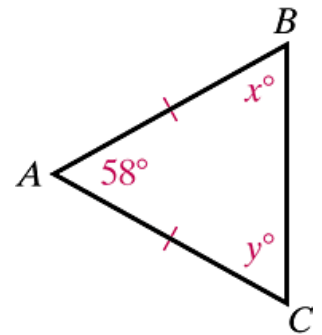


Then...



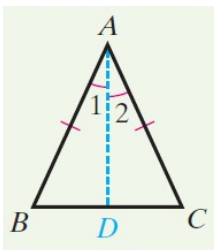
Example Using Isosceles Triangles

Use the figure and markings to find the values of x and y .

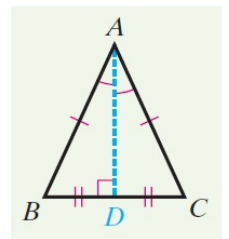


Theorem Perpendicular Bisector of the Base of an Isosceles Triangle

If...

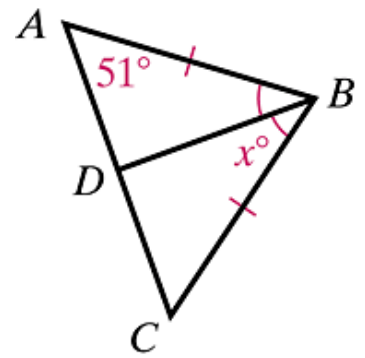


Then...



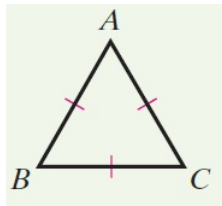
Example Using Isosceles Triangles

Use the given figure to find the value of x .

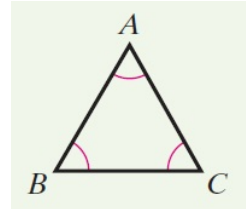


Corollary If Equilateral than Equiangular Triangle

If...

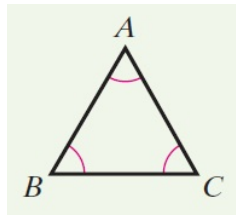


Then...

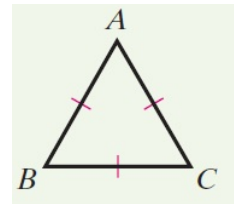


Corollary If Equiangular then Equilateral Triangle

If...

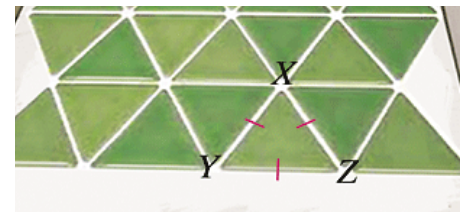


Then...



Example Using Equilateral Triangles

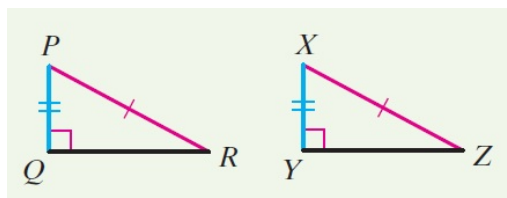
Each triangle in the floor pattern shown is an equilateral triangle. Find the measure of each angle in $\triangle XYZ$.



Hypotenuse-Leg (H-L) Theorem

If...

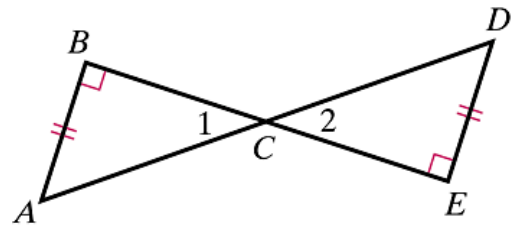
Then...



Example Using the H-L Theorem

Given: \overline{BE} bisects \overline{AD} at C
 $\overline{AB} \perp \overline{BC}, \overline{DE} \perp \overline{EC}$
 $\overline{AB} \cong \overline{DE}$

Prove: $\triangle ABC \cong \triangle DEC$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.