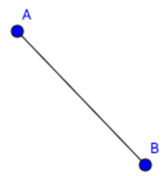
MAT 222

Chapter 5: Things To Know

Section 5.1 Perpendicular and Angle Bisectors

Objectives	**Vocabulary**
1. Use Perpendicular Bisectors to Solve Problems. 2. Use Angle Bisectors to Solve Problems.	• equidistant • distance from a point to a line

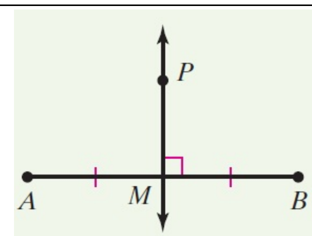
Review Constructing Perpendicular Bisectors
Construct the bisector of the segment below.



Perpendicular Bisector Theorem

If…

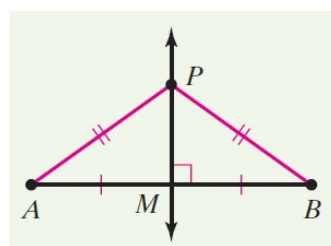
Then…



Converse of Perpendicular Bisector Theorem

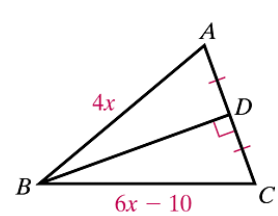
If…

Then…



Looking at the picture above, how do you think these theorems would be proved?

Example Using the Perpendicular Bisector Theorem
Use the given figure to find the length of $\overline{AB}$.



Example Using the Perpendicular Bisector Theorem to Solve an Application

A chef wants to open a new Japanese restaurant in the city of Metropolis. Currently, the city has two successful Japanese restaurants, so the chef is looking for a location within the city limits, but one that is equidistant, and as far away as possible, from the two current restaurants.

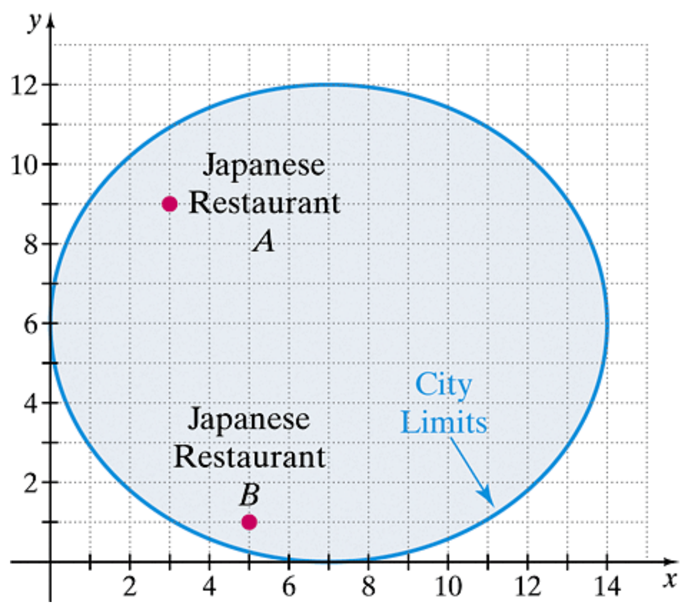
a. Explain the process for finding the location the chef is looking for.

b. Find the coordinates of the midpoint of $\overline{AB}$; call it point C.

c. Find the slope of $\overline{AB}$, and then the slope of the line perpendicular to $\overline{AB}$.

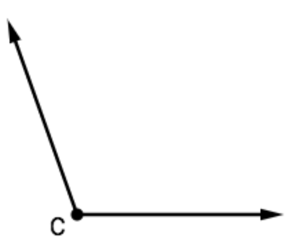
d. Locate point C and use the perpendicular slope from part **c** to draw the perpendicular bisector of $\overline{AB}$.

e. Approximate the coordinates of the chef's desired location for the new restaurant. (Use whole number coordinates within the city limits.)



Review Constructing Angle Bisectors

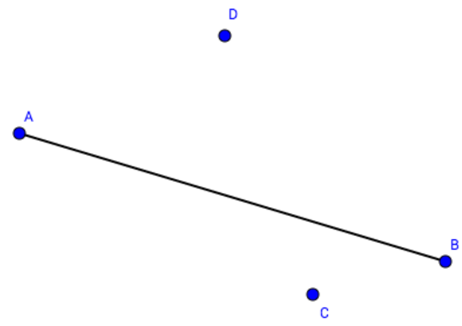
Construct the bisector of the angle below.



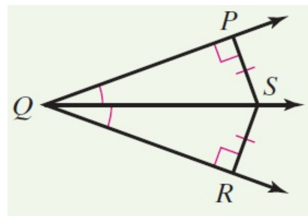
Definition

The distance from a point to a line is ______________________________________.

Draw the distance from the points C and D to $\overline{AB}$ below.



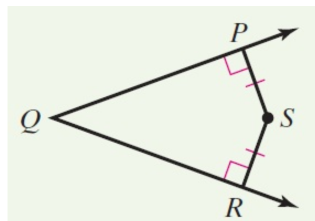
Angle Bisector Theorem



If…

Then…

Converse of the Angle Bisector Theorem

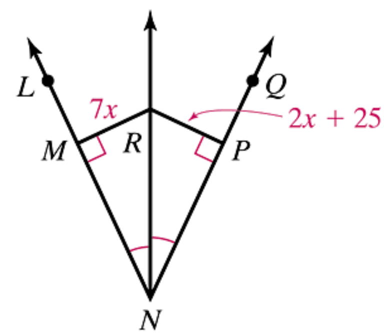


If…

Then…

Example Using the Angle Bisector Theorem

Use the given figure and find the length of $\overline{RM}$.



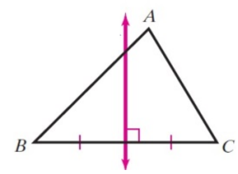
Section 5.2 Bisectors of a Triangle

Objectives	**Vocabulary**
1. Use Properties of the Perpendicular Bisectors of the Sides of a Triangle, Including the Circumcenter. 2. Use Properties of the Angle Bisectors of the Angles of a Triangle, Including the Incenter.	• concurrent • point of concurrency • circumcenter of a triangle • circumscribed about • incenter of a triangle • inscribed in

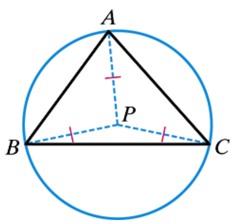
Definitions

A **perpendicular bisector of a triangle** is a line that is ____________________

to the side of the triangle at the triangle's ____________________.



When three or more lines intersect at one point, they are ____________________.



We say the circle to the left is ____________________ ____________

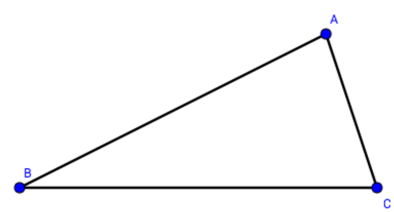
the triangle.

Theorem Concurrency of Perpendicular Bisectors

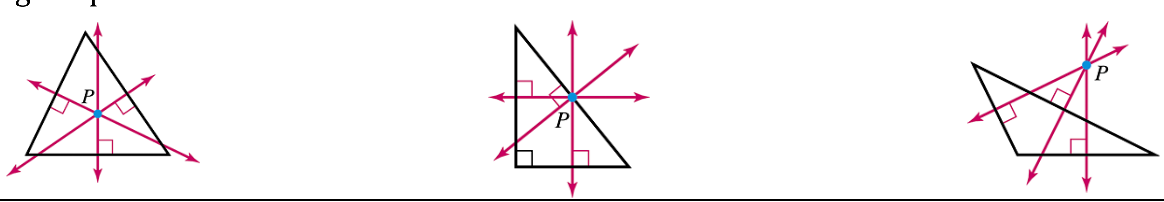
The perpendicular bisectors of the sides of a triangle are concurrent.

Their point of concurrency, P, is called the ________________________ of the triangle.

Illustrate the theorem by sketching in ***the perpendicular bisectors of all three sides of this triangle, as well as the circle*** *with center P that passes through all three vertices of the triangle.*

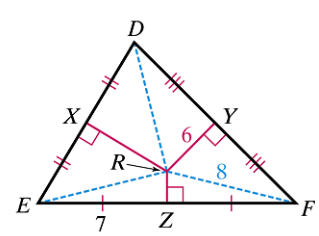


Describe how the circumcenter is related to the type of triangle using the pictures below.



Example Using the Circumcenter of a Triangle

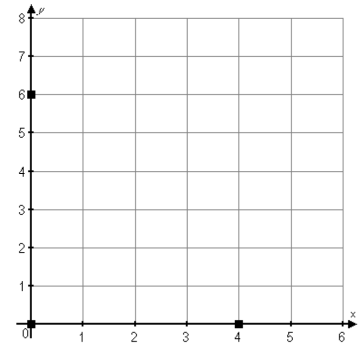
The circumcenter of ΔDEF is point R. Fill in the blanks.



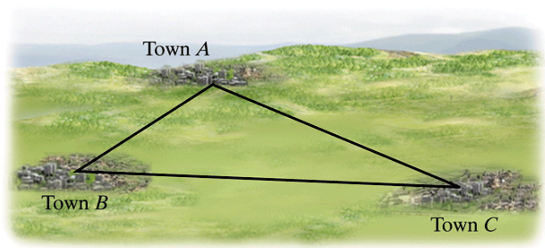
a. $RD =$ ________ $=$ ________

b. $RD =$ ________ units

Example Finding the Circumcenter of a Triangle
What are the coordinates of the circumcenter of the triangle with vertices $P(0,6)$, $O(0,0)$, and $S(4,0)$?



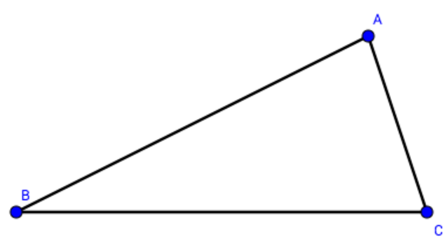
Example Using the Circumcenter in an application
A recycling center is to be built to service three neighboring towns. To save fuel, the center is to be built equidistant from towns A, B, and C. Where should the center be built?



Theorem Concurrency of Angle Bisectors
The angle bisectors of a triangle are concurrent.

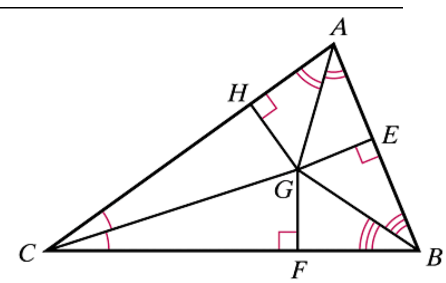
Their point of concurrency, P, is called the ______________________________ of the triangle.

Illustrate the theorem by sketching in ***the angle bisectors of all three angles of this triangle.***



The incenter of the triangle is also the center of the circle that is _________________________ ________ the triangle. *Illustrate this by sketching this circle in your picture above.*

Example Identifying and Using the Incenter of a Triangle
$GE = 2x - 7$ and $GF = x + 4$. What is GH?

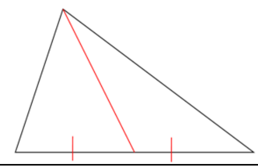


Section 5.3 Medians and Altitudes of a Triangle

Objectives	**Vocabulary**
1. Use Properties of the Medians of a Triangle. 2. Use Properties of the Altitudes of a Triangle.	• median of a triangle • centroid of a triangle • altitude of a triangle • orthocenter of a triangle

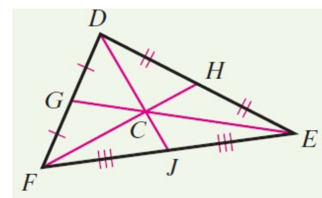
Definition
The median of a triangle is a segment whose endpoints are a ______________________________

and the ____.



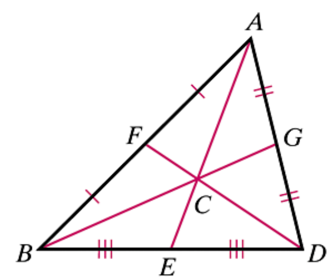
Theorem Concurrency of Medians
The three medians of a triangle are concurrent.

The point of concurrency of the medians is _________________________
the distance from each vertex to the midpoint of the opposite side.



Equations: __________________ , __________________ , __________________

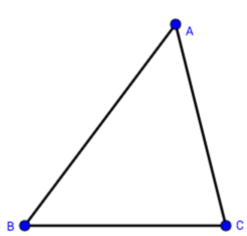
Example Finding the Length of a Median
In the diagram, $AC = 10$ units. Find AE.

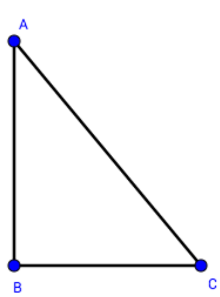


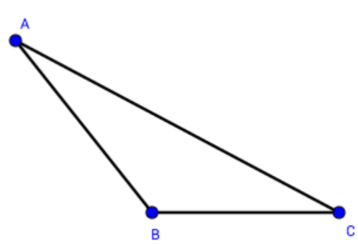
Definition
The altitude of a triangle is the __________________________ segment from _____________________ of the

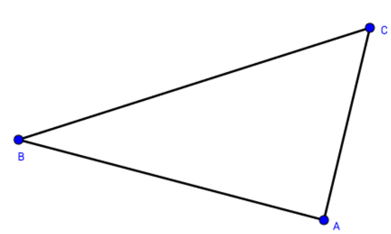
triangle to ____.

Illustrate this definition by sketching the altitudes from the vertex A to the opposite side in each triangle.









Example Identifying Medians and Altitudes

For *PQS*, identify the given segments as a median, an altitude, or neither.

a. $\overline{QT}$

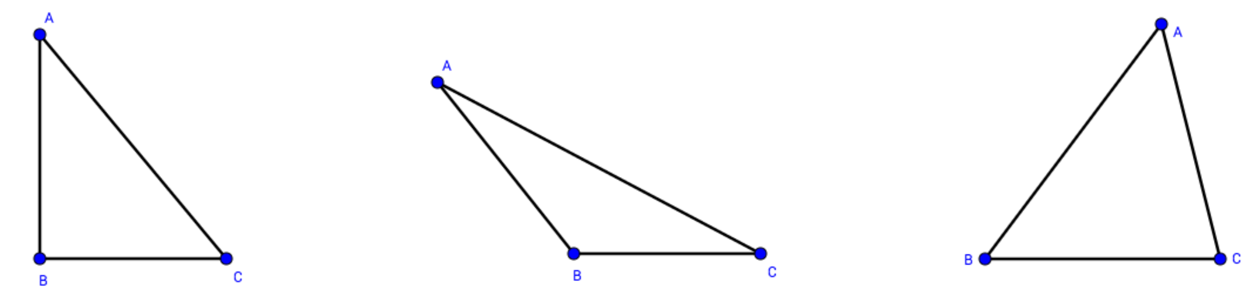
b. $\overline{PR}$

Theorem Concurrency of Altitudes

The lines that contain the altitudes of a triangle are concurrent.

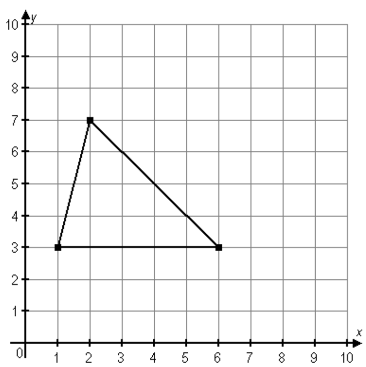
The point of concurrency of the altitudes of a triangle is called its ___________________________________.

Illustrate the theorem by drawing all of the altitudes of each triangle below. Label each triangle as "acute," "right," or "obtuse," and label each orthocenter as "inside," "on," or "outside" the triangle.



Example Finding the Orthocenter of a Triangle

ΔABC has vertices $A(1,3), B(2,7)$ and $C(6,3)$. What are the coordinates of the orthocenter of the triangle?

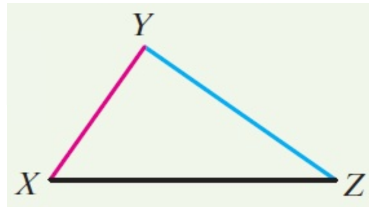


Note: We are skipping Section 5.4 and part of Section 5.5 (deleted from 5.5 PowerPoint).

Section 5.5 Inequalities in One Triangle

Objectives	**Vocabulary**
1. Use the Triangle Inequality Theorem	• No new vocabulary

Triangle Inequality Theorem
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



Inequalities:

Example Using the Triangle Inequality Theorem
Can a triangle have sides with the given lengths? Explain.

a. 3 ft, 7 ft, 8 ft

b. 5 ft, 10 ft, 15 ft

Example Finding Possible Side Lengths
Two sides of a triangle are 5 ft and 8 ft long. What is the range of possible lengths for the third side?

Note: We are skipping Section 5.6.