MAT-222-840

Spring 2017

Name: Solutions

Date: ______________________

Team Problems for Chapter 7

Problem #1: The Capture-Recapture Method

To determine the number of deer in a forest, a forest ranger tags 280 of them and releases them back into the forest. Later, 405 deer are caught, out of which 45 of them are tagged. Set up and solve a proportion to estimate how many deer are in the forest.

Let x = total # of deer in forest

$$\frac{45}{405} = \frac{280}{x}$$

$$(\cancel{405}x)\frac{45}{\cancel{405}} = \frac{280}{\cancel{x}}(405\cancel{x})$$

$$\frac{\cancel{45}x}{\cancel{45}} = \frac{280 \cdot 405}{45}$$

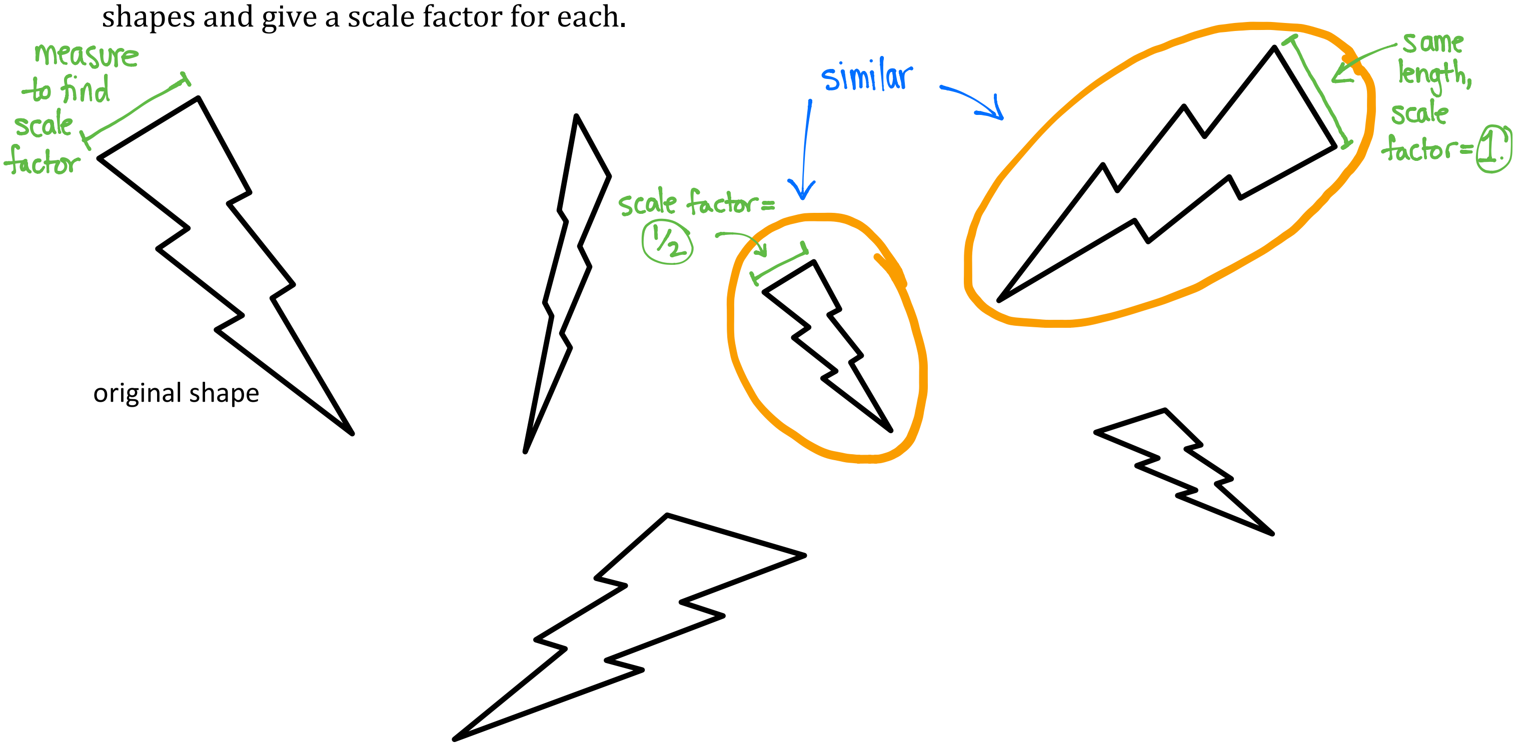
$$x = \frac{113400}{45} = 2520$$

Approximately 2520 deer

Problem #2: Mathematical Similarity versus Similarity in Everyday Language

In mathematics, the terms similar and similarity have a much more specific meaning than they do in everyday language.

Examine the shapes that follow. In everyday language, we might say that all the shapes are similar. However, mathematically speaking, only two of them are. Identify the mathematically similar shapes and give a scale factor for each.



Sometimes a problem that looks as if it could be solved by setting up a proportion actually can't be solved that way. Before you set up a proportion to solve a problem, ask the following question about quantities in the problem: If I double one of the quantities, should the other quantity also double? If the answer is no, then you cannot solve the problem by setting up a proportion.

Problem #3: Can you always use a proportion? (Part 1)

Read the text in the box above, then answer the following.

Ken used 3 loads of stone pavers to make a circular (i.e., circle-shaped) patio with a radius of 10 feet. Ken wants to make another circular patio with a radius of 15 feet, so he sets up the proportion

$$\frac{3 \text{ loads}}{10 \text{ feet}} = \frac{x \text{ loads}}{15 \text{ feet}}$$

Is this correct? If not, why not? Is there another proportion that Ken could set up to solve the problem? (The area of a circle that has radius r units is πr^2 square units.)

Since the area is proportional to r^2, not r, we must set up the proportion as follows:

$$\frac{3}{10^2} = \frac{x}{15^2}$$

Ken will need about 7 loads of pavers.

$$\frac{3}{100} = \frac{x}{225}$$

$$x = 6.75$$

Problem #4: Can you always use a proportion? (Part 2)

Read the text in the box above, then answer the following.

In a cookie factory, 4 assembly lines make enough boxes of cookies to fill a truck in 10 hours. How long will it take to fill the truck if 8 assembly lines are used? Is the proportion

$$\frac{10 \text{ hours}}{4 \text{ lines}} = \frac{x \text{ hours}}{8 \text{ lines}}$$

appropriate for this situation? Why or why not? If not, can you solve the problem another way? (Assume that all assembly lines work at the same steady rate.)

No: this proportion would indicate that it takes more time to fill a truck with more workers, which doesn't make sense.

Instead, note that twice as many workers would fill the truck in half the time, 5 hours.

Problem #5: Using Scaling to Understand Astronomical Distances

Ms. Frizzle's class has been studying planets and stars. Ms. Frizzle wants to help the students get a better sense of astronomical distances by scaling down these distances. The table that follows shows the distances in kilometers from the sun to the earth, the sun to Pluto, and the sun to Alpha Centauri. Alpha Centauri is one of the closest stars to the sun.

Heavenly Body	Approximate Distance from Sun
Earth	150 million km
Pluto	5.9 billion km
Alpha Centauri	38 trillion km

If Ms. Frizzle represents the distance from the earth to the sun as 10 centimeters (about the width of a hand), then how should she represent the distance from Pluto to the sun?

$$\frac{10 \text{ cm}}{x \text{ cm}} = \frac{150 \text{ million km}}{5.9 \text{ billion km}}$$

$$x \text{ cm} = \frac{10 \cdot 5{,}900{,}000{,}000}{150{,}000{,}000} = 393.\overline{3} \text{ cm} \approx 4 \text{ meters}$$, a little less than the length of a car.

How should she represent the distance from Alpha Centauri to the sun?

$$\frac{10 \text{ cm}}{x \text{ cm}} = \frac{150 \text{ million km}}{38 \text{ trillion km}}$$

$$x \text{cm} = \frac{10 \cdot 38000000000000}{150000000} = 2533333.\overline{3} \text{ cm} = 25333.\overline{3} \text{ meters} = 25.\overline{3} \text{ km}$$

This is approximately the distance between AACC's Arnold & Arundel Mills campuses.

Will Ms. Frizzle be able to show these distances in her classroom?

Ms. Frizzle's classroom might be large enough to show the distance from the sun to Pluto, but it is definitely too small to show the distance from our sun to Alpha Centauri.