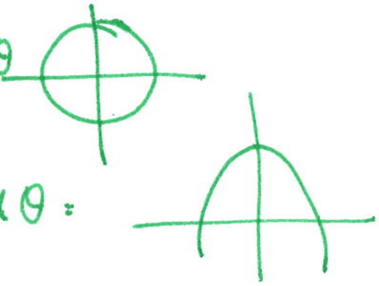
Math 240, Quiz #10, Summer 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Evaluate the integral $\int_0^{2\pi}\int_0^{\sqrt{3}}\int_0^{3-r^2} r\,dz\,dr\,d\theta$. Sketch or describe the region.

$$\int_0^{2\pi}\int_0^{\sqrt{3}} rz\Big|_0^{3-r^2} dr\,d\theta = \int_0^{2\pi}\int_0^{\sqrt{3}} r(3-r^2)\,dr\,d\theta$$



$$= \int_0^{2\pi}\int_0^{\sqrt{3}} 3r - r^3\,dr\,d\theta = \int_0^{2\pi} \frac{3}{2}r^2 - \frac{1}{4}r^4\Big|_0^{\sqrt{3}} d\theta =$$

$$\int_0^{2\pi} \frac{3}{2}(3) - \frac{1}{4}(9)\,d\theta = \int_0^{2\pi} \frac{9}{2} - \frac{9}{4}\,d\theta = \int_0^{2\pi} \frac{9}{4}\,d\theta =$$

$$\frac{9}{4}\theta\Big|_0^{2\pi} = \frac{9}{4}(2\pi) = \boxed{\frac{9\pi}{2}}$$

2. Convert the integral $\int_0^5\int_0^{\sqrt{25-x^2}}\int_0^{\sqrt{25-x^2-y^2}} \frac{1}{1+x^2+y^2+z^2}\,dz\,dy\,dx$ into spherical coordinates and use that to evaluate it.

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^5 \frac{1}{1+\rho^2}\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

$$\frac{\rho^2}{1+\rho^2} = \begin{array}{r} 1 \\ 1+\rho^2\overline{)\,\rho^2\quad} \\ \underline{-\rho^2 \mp 1} \\ -1 \end{array}$$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^5\left(1 - \frac{1}{1+\rho^2}\right)d\rho\,\sin\varphi\,d\varphi\,d\theta = \int_0^{\pi/2}\int_0^{\pi/2}(\rho - \arctan\rho\Big|_0^5 \sin\varphi\,d\varphi\,d\theta$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}(5-\arctan 5)\sin\varphi\,d\varphi\,d\theta = \int_0^{\pi/2} -(5-\arctan 5)\underset{-0+1}{\cos\varphi\Big|_0^{\pi/2}}\,d\theta =$$

$$\int_0^{\pi/2}(5-\arctan 5)\,d\theta = \boxed{\frac{\pi}{2}(5-\arctan 5)}$$