Math 240, Quiz #11, Summer 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Find the integrals needed to calculate the center of mass for the region bounded by the surfaces, and with the given density $z = 4 - x,\ x = 0,\ x = 4,\ y = 0,\ y = 4,\ z = 0,$ $\rho(x, y, z) = ky$

$$M = \int_0^4 \int_0^4 \int_0^{4-x} ky \, dz\, dy\, dx$$

$$M_{yz} = \int_0^4 \int_0^4 \int_0^{4-x} kxy \, dz\, dy\, dx$$

$$\int_0^4 \int_0^4 \int_0^{4-x} ky^2 \, dz\, dy\, dx = M_{xz}$$

$$M_{xy} = \int_0^4 \int_0^4 \int_0^{4-x} kyz \, dz\, dy\, dx$$

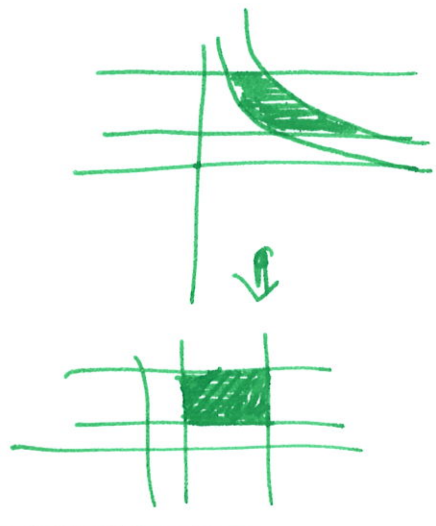
2. For the function $f(x, y) = y\sin(xy)$, calculate the ~~region~~ volume below it on the region bounded by $xy = 1, y = 4, xy = 4, and\ y = 1$. Do this by changing to a convenient pair of variables. Sketch the region before and after the switch.

$y = \frac{1}{x}$ $y = 4$ $y = \frac{4}{x}$ $y = 1$

$u = xy$ $v = y$ $u \in [1, 4]$

$u = x \cdot v \Rightarrow x = \frac{u}{v}$ $v \in [1, 4]$

$$J = \begin{vmatrix} \frac{1}{v} & \frac{-u}{v^2} \\ 0 & 1 \end{vmatrix} = \frac{1}{v}$$



$$\int_1^4 \int_1^4 \cancel{v} \sin u \cdot \cancel{\tfrac{1}{v}} \, du\, dv = \int_1^4 \int_1^4 \sin u \, du\, dv$$

$$\int_1^4 -\cos u \Big|_1^4 dv = \int_1^4 (\cos 1 - \cos 4)\, dv = \boxed{3(\cos 1 - \cos 4)}$$