Math 240, Quiz #14, Summer 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Calculate the flow through the closed surface using the Divergence Theorem.

$$\vec{F}(x,y,z) = (x+y)\hat{\imath} + y\hat{\jmath} + z\hat{k}, S: z = 16 - x^2 - y^2, z = 0$$

$\vec{\nabla}\cdot\vec{F} = 1+1+1 = 3$

$x^2+y^2=16$

$r=4$

$\int_0^{2\pi}\int_0^4\int_0^{16-r^2} 3r\,dz\,dr\,d\theta$

$= \int_0^{2\pi}\int_0^4 3r(16-r^2)\,dr\,d\theta = \int_0^{2\pi}\int_0^4 48r - 3r^3\,dr\,d\theta$

$\int_0^{2\pi} 24r^2 - \frac{3}{4}r^4\Big|_0^4 d\theta = \int_0^{2\pi} 192\,d\theta = 384\pi$

2. Use Stokes' Theorem to evaluate the line integral over the surface.

$$\vec{F}(x,y,z) = x^2\hat{\imath} + z^2\hat{\jmath} - xyz\hat{k}, S: z = \sqrt{4-x^2-y^2}$$

$x^2+y^2=4$ $r=2$

$\langle 0,0,1\rangle = \hat{k} = \hat{n}$

$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & z^2 & -xyz \end{vmatrix} =$

$(-xz-2z)\hat{\imath} - (-yz)\hat{\jmath} + (0-0)\hat{k}$

$\vec{\nabla}\times\vec{F}\cdot\vec{n} = \vec{0}$

$\int_0^{2\pi}\int_0^2 0\cdot r\,dr\,d\theta = 0$