Math 240, Quiz #4, Summer 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. If a plane is flying toward a target at 36,000 feet at 600 miles per hour, and it wants to drop a bomb on that target whose map coordinates are given by (0,0). What map coordinates should the plane be above in order for the bomb to hit the target when it lands?

$600t\,\hat{i}$

$\left(-\frac{1}{2}gt^2 + 36000\right)\hat{j}$

$-16t^2 + 36000 = 0$

$t^2 = 2250$

$t = \pm 15\sqrt{10} \approx \pm 47.43$

drop -47.43 seconds (before passing target) or about 28,460.5 feet before target or 5.39 miles out

2. Find the curvature of the functions

   a. $\vec{r}(t) = t^2\hat{\imath} + \cos(t)\,\hat{\jmath} + \sin(2t)\,\hat{k}$

$r' = 2t\,\hat{i} - \sin t\,\hat{j} + 2\cos 2t\,\hat{k}$

$r'' = 2\,\hat{i} - \cos t\,\hat{j} - 4\sin 2t\,\hat{k}$

$$r' \times r'' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2t & -\sin t & 2\cos 2t \\ 2 & -\cos t & -4\sin 2t \end{vmatrix} =$$

$(4\sin t \sin 2t + 2\cos t \cos 2t)\,\hat{i}$

$-(8t\sin 2t - 4\cos 2t)\,\hat{j}$

$+(-2t\cos t + 2\sin t)\,\hat{k}$

$$K = \frac{\sqrt{(4\sin t \sin 2t + 2\cos t\cos 2t)^2 + (4\cos 2t - 8t\sin 2t)^2 + (2\sin t - 2t\cos t)^2}}{\left(\sqrt{(2t)^2 + \cos^2 t + 16\sin^2 2t}\right)^3}$$

   b. $y = x^3 + x$

$y' = 3x^2 + 1$

$y'' = 6x$

$$K = \frac{|6x|}{(3x^2+1)^{3/2}}$$