Math 240, Quiz #6, Summer 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Find $\lim_{(x,y)\to(0,0)} \frac{2x^3y^2}{x^6+y^4}$ if it exists, or prove that it does not.

$x^6 = y^4$

$x^3 = y^2$

$y = x^{3/2}$

path $y = kx^{3/2}$

$$\lim_{x\to 0} \frac{2x^3(kx^{3/2})^2}{x^6+(kx^{3/2})^4} = \lim_{x\to 0} \frac{2k^2\cancel{x^6}}{\cancel{x^6}(1+k^4)} = \lim_{x\to 0} \frac{2k^2}{1+k^4} = \frac{2k^2}{1+k^4}$$

this value depends on k, so DNE

2. Find the equations of the tangent plane and the normal line to the curve $xyz=10, P(1,2,5)$ at the given point.

$F = xyz - 10$

$\nabla F = \langle yz, xz, xy\rangle \Rightarrow \langle 10, 5, 2\rangle$

normal line $\vec{r}(t) = (10t+1)\hat{\imath} + (5t+2)\hat{\jmath} + (2t+5)\hat{k}$

tangent plane $10(x-1)+5(y-2)+2(z-5)=0$

3. Find $\frac{\partial w}{\partial t}$ for the following sets of equations using the chain rule. Be sure your final answers contain only t.

$$w = x^2+y^2, x = s+t, y = s-t$$

$\frac{\partial w}{\partial x} = 2x$ $\qquad \frac{\partial w}{\partial y} = 2y$

$\frac{\partial x}{\partial t} = 1$ $\qquad \frac{\partial y}{\partial t} = -1$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial t} = 2x(1) + 2y(-1) =$$

$$2(s+t) - 2(s-t)$$

$$= 2s + 2t + 2s - 2t$$

$$= 4t$$