

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Use the method of Lagrange multipliers to maximize the function subject to the given constraint(s) $w = x^2 - 10x + y^2 - 14y + 28$, subject to $x + y = 10$

$$\nabla w = \lambda \nabla g$$

$$-10 + 2x = \lambda$$

$$2y - 14 = \lambda$$

$$-10 + 2x = 2y - 14$$

$$2x - 2y = -4$$

$$x - y = -2$$

$$w = 4^2 - 10(4) + 6^2 - 14(6) + 28 = \boxed{36}$$

$$\begin{aligned} g &= x + y - 10 = 0 \\ x + y + 2 &= 0 \end{aligned}$$

$$2x - 8 = 0$$

$$2x = 8$$

$$x = 4$$

$$x - y = -2$$

$$4 - y = -2$$

$$-y = -6$$

$$(4, 6)$$

$$y = 6$$

2. Set up a double integral and use it to evaluate the volume over the region R . $\iint_R \sin x \sin y dA$;

R : rectangle with vertices $(-\pi, 0), (\pi, 0), (\pi, \frac{\pi}{2}), (-\pi, \frac{\pi}{2})$. Sketch the region in the plane.

$$\int_0^{\frac{\pi}{2}} \int_{-\pi}^{\pi} \sin x \sin y dx dy =$$

$$\int_0^{\frac{\pi}{2}} \sin y dy \left[-\cos x \right]_{-\pi}^{\pi} = \int_0^{\frac{\pi}{2}} [(-1) + (-1)] \sin y dy$$

$$= \int_0^{\frac{\pi}{2}} 0 \cdot \sin y dy = \boxed{0}$$