MTH 174 Practice Exam #4 Key

1. a. $f(x) = \dfrac{e^x - 1}{x}$ $\qquad e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}$ $\qquad e^x - 1 = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} - 1 = \sum_{n=1}^{\infty} \dfrac{x^n}{n!}$

$$\frac{e^x - 1}{x} = \frac{\sum_{n=1}^{\infty} \frac{x^n}{n!}}{x} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!}$$

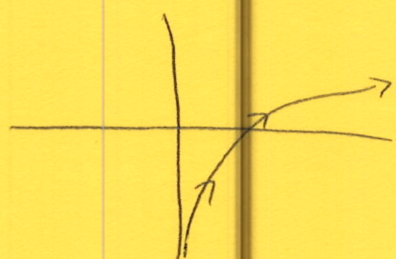
b. $f(x) = \int \cos x^2 \, dx$ $\qquad \cos x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{(2n)!}$ $\qquad \cos x^2 = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{4n}}{(2n)!}$

$$\int \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(4n+1)(2n)!}$$

2. a.

t	x	y
0	0	$-\infty$
1	1	0
2	8	$3\ln 2$
3	27	$3\ln 3$
4	64	$3\ln 4$

b.



c. $x = t^3 \qquad y = \ln t^3$

$\Rightarrow y = \ln x$

d. $\dfrac{dx}{dt} = 3t^2 \qquad \dfrac{dy}{dt} = \dfrac{3}{t} \qquad \dfrac{dy}{dx} = \dfrac{\frac{3}{t}}{3t^2}$

$= \dfrac{3}{3t^3} = \dfrac{1}{t^3}$

e. $\dfrac{dy}{dx}(1) = \dfrac{1}{1} = 1$

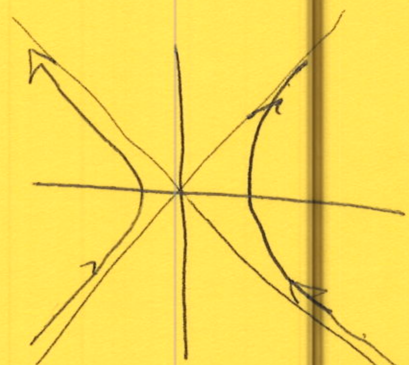
$y - 0 = 1(x-1) \Rightarrow$

$y = x - 1$

3. a.

t	x	y
0	1	0
$\pi/6$	$\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
$\pi/4$	$\sqrt{2}$	1
$\pi/3$	2	$\sqrt{3}$
$\pi/2$	∞	∞

b.



d. $\dfrac{dx}{dt} = \sec t \tan t$

$\dfrac{dy}{dt} = \sec^2 t$

$\dfrac{dy}{dx} = \dfrac{\sec^{\cancel{2}} t}{\cancel{\sec} t \tan t}$

$= \sec t \cot t$

$= \dfrac{1}{\cancel{\cos t}} \cdot \dfrac{\cancel{\cos t}}{\sin t} =$

$= \csc t$

c. $1 + \tan^2 t = \sec^2 t$

$1 + y^2 = x^2 \rightarrow x^2 - y^2 = 1$

e. $0, \pi$, etc.

$k\pi \rightarrow$ vertical

never horizontal

4. $\Delta x = 5 - 1 = 4$

$\Delta y = -2 - 4 = -6$

$x = 4t + 1 \qquad y = 4 - 6t$

5. $\frac{dx}{dt} = 3 - 2t \qquad \frac{dx}{dt}(1/4) = 3 - \frac{1}{2} = \frac{5}{2}$

$\frac{dy}{dt} = \cancel{2 \cdot \frac{3}{2}} t^{1/2} = 3t^{1/2} \qquad \frac{dy}{dt}(1/4) = 3\sqrt{1/4} = \frac{3}{2}$

$\frac{dy}{dx}(1/4) = \frac{3/2}{5/2} = \frac{3}{\cancel{2}} \cdot \frac{\cancel{2}}{5} = \frac{3}{5}$

$x(\frac{1}{4}) = 3(1/4) - (\frac{1}{4})^2 = \frac{11}{16}$

$y(\frac{1}{4}) = 2(\frac{1}{4})^{3/2} = 2(\frac{1}{8}) = \frac{1}{4}$

$y - \frac{1}{4} = \frac{3}{5}\left(x - \frac{11}{16}\right)$

6. $r = 3\cos 2\theta = 0$

$\cos 2\theta = 0$

$2\theta = \pi/2,\ 3\pi/2,\ 5\pi/2,\ \frac{7\pi}{2}$

$\theta = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4$

$y = r\sin\theta \qquad x = r\cos\theta$

$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$

$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} = \frac{-6\sin 2\theta\sin\theta + 3\cos 2\theta\cos\theta}{-6\sin 2\theta\cos\theta - 3\cos 2\theta\sin\theta}$$

$$\frac{dy}{dx}(@\ \theta = \pi/4) = \frac{-6\sin(\pi/2)\sin(\pi/4) + \cancel{3\cos(\pi/2)\cos(\pi/4)}}{-6\sin(\pi/2)\cos(\pi/4) - \cancel{3\cos(\pi/2)\sin(\pi/4)}} \to 0$$

$$= \frac{-6(1)(\frac{1}{\sqrt{2}})}{-6(1)\frac{1}{\sqrt{2}}} = 1 \qquad y = x$$

$$\frac{dy}{dx}(@\ \theta = 3\pi/4) = \frac{-6\sin(3\pi/2)\sin(3\pi/4) + \cancel{3\cos(3\pi/2)\cos(3\pi/4)}}{-6\sin(3\pi/2)\cos(3\pi/4) - \cancel{3\cos(3\pi/2)\sin(3\pi/4)}} \to 0 =$$

$$\frac{-6(-1)(\frac{1}{\sqrt{2}})}{-6(-1)(-\frac{1}{\sqrt{2}})} = -1 \qquad y = -x$$

7. $A = \frac{1}{2}\int_{-\pi/6}^{\pi/6} (4\cos 3\theta)^2\, d\theta = 16\int_0^{\pi/6} \cos^2 3\theta\, d\theta = 8\int_0^{\pi/6} 1+\cos 6\theta\, d\theta$

$= 8\left[\theta + \frac{1}{6}\cancel{\sin 6\theta}^{\,0}\right|_0^{\pi/6} = 8\cdot\frac{\pi}{6} = \frac{4\pi}{3}$

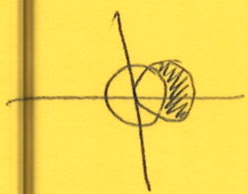
8.

$1 = 2\cos\theta$

$\frac{1}{2} = \cos\theta$

$\theta = \pm \pi/3$

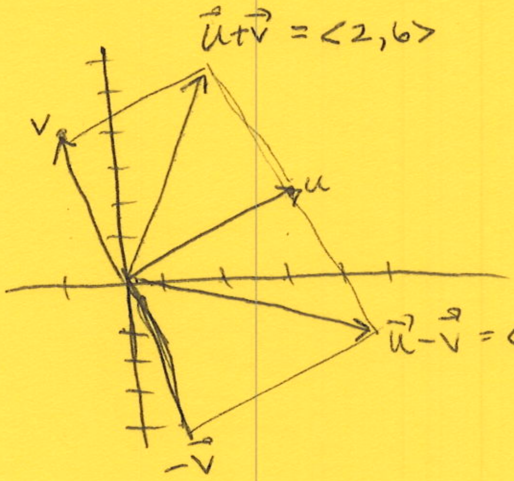
$\frac{1}{2}\int_{-\pi/3}^{\pi/3} (2\cos\theta)^2 - 1^2\, d\theta$



$\int_0^{\pi/3} 4\cos^2\theta - 1\, d\theta = \int_0^{\pi/3} 2(1+\cos 2\theta) - 1\, d\theta =$

$\int_0^{\pi/3} 1 + 2\cos 2\theta\, d\theta = \theta + \cancel{\sin 2\theta}\Big|_0^{\pi/3}$ $\pi/3 + \sin 2\pi/3 = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

9.



The resulting $\vec{u}+\vec{v}$ vector is the diagonal of the parallelogram defined by $\vec{u}$ and $\vec{v}$.

likewise for $\vec{u}$ and $(-\vec{v})$

10. $F_1 = \langle 75\cos 30°, 75\sin 30°\rangle = \langle \frac{75\sqrt{3}}{2}, \frac{75}{2}\rangle$

$F_2 = \langle 100\cos 45°, 100\sin 45°\rangle = \langle 50\sqrt{2}, 50\sqrt{2}\rangle$

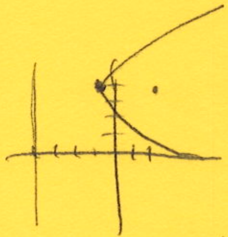
$F_3 = \langle 125\cos 120°, 125\sin 120°\rangle = \langle -\frac{125}{2}, \frac{125\sqrt{3}}{2}\rangle$

$= \langle 73.16, 216.46\rangle$

F_{TOTAL}

$\|F_{TOTAL}\| = \sqrt{73.16^2 + 216.46^2} = 228.49$

$\theta_T = \tan^{-1}\left(\frac{216.46}{73.16}\right) = 71.3°$

11. 

$\frac{2-(-4)}{2} = \frac{6}{2} = 3 = a$

vertex $(-1,4)$

$$(y-4)^2 = 4(3)(x+1)$$

$$(y-4)^2 = 12(x+1)$$

12. a. $r = 4\sin 2\theta$

b. $r = 2 - 2\cos\theta$

13. a. $\vec{u} + \vec{v} = \langle 16, -2 \rangle$

b. $\|\vec{u}\| = \sqrt{16+9} = \sqrt{25} = 5$

c. $\|\vec{v}\| = \sqrt{144+25} = \sqrt{169} = 13$

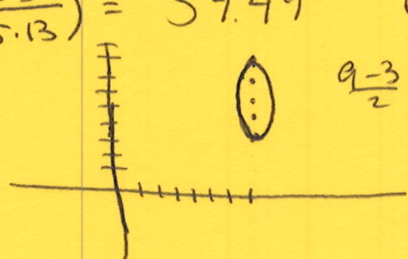
$\theta = \tan^{-1}\left(\frac{-5}{12}\right) = -.3948$ radians $(-22.62°)$

$\vec{v} = \langle 13\cos(-.3948), 13\sin(-.3948) \rangle$

d. $\frac{\vec{u}}{\|\vec{u}\|} = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$

e. $48 - 15 = 33$

f. $\theta = \cos^{-1}\left(\frac{33}{5 \cdot 13}\right) = 59.49°$ or 1.04 radians

14. 

$\frac{9-3}{2} = \frac{6}{2} = 3 = a$

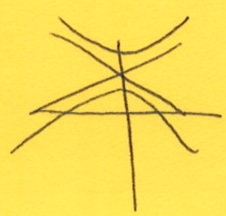
$c = 2$

$3^2 - 2^2 = b^2$

$9 - 4 = 5$ $\quad b = \sqrt{5}$ $\qquad$ center $(7,6)$

$$\frac{(x-7)^2}{9} + \frac{(y-6)^2}{5} = 1$$

15. $r = \frac{4}{1 + 2\sin\theta}$ $\qquad e = 2$ hyperbola $e > 1$



16. $r = 1 - 2\cos\theta$ use polar graph paper

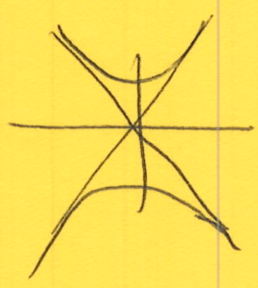
θ	r
0	-1
$\pi/6$	$-.7321$
$\pi/4$	$-.4142$
$\pi/3$	0
$\pi/2$	1
$2\pi/3$	2

17. $r = \dfrac{1}{1 - 2\cos\theta}$

θ	r
0	-1
$\pi/6$	-1.366
$\pi/4$	-2.414
$\pi/3$	error
$\pi/2$	1
$2\pi/3$	$1/2$

18. $a = 4$ $c = 5$ $5^2 - 4^2 = 9 \Rightarrow b = 3$

$$\frac{y^2}{16} - \frac{x^2}{9} = 1$$



19. $a = 5$

$e = \dfrac{c}{a} = \dfrac{2}{5}$ $c = 2$ $(-2, 3)$

$25 - 4 = 21$ $b = \sqrt{21}$

$$\frac{(x+2)^2}{25} + \frac{(y-3)^2}{21} = 1 \quad \text{or} \quad \frac{(x+2)^2}{21} + \frac{(y-3)^2}{25} = 1$$

one of these