First-order application problems, Solutions.

Directions: Solve each of the first-order application problems. Submit your problems to me by email and I will apply up to 7 points per problem to your Exam #1 grade. (The total of your Exam #1 grade and these points cannot exceed 100.)

1. A tank is filled with 500 gallons of brine solution containing 5 pounds of salt at the outset. A brine solution containing 10 ounces of salt per gallon flows into the tank at the rate of 4 gallons per minute. The well-mixed solution flows out of the tank at the same rate. Set up and solve the system to determine the amount of salt in the tank after 2 hours.

$$R_{in} = \frac{10\ oz.}{gal} \times \frac{4\ gal}{\text{min.}} = 40, \qquad R_{out} = \frac{A}{500\ gal} \times \frac{4\ gal}{min} = \frac{A}{125}$$

$$\frac{dA}{dt} = 40 - \frac{A}{125} = -\frac{1}{125}(A - 5000) \rightarrow \frac{dA}{A - 5000} = -\frac{1}{125}dt$$

$$\ln|A - 5000| = -\frac{1}{125}t + C \rightarrow A(t) = A_0 e^{-\frac{1}{125}t} + 5000$$

$$A(0) = 80, 80 = A_0 + 5000 \rightarrow A(t) = 5000 - 4920e^{-\frac{1}{125}t}$$

$$A(120) = 3116.17\ oz.$$

Part 2. If the outflow from the tank is 5 gallons per minute, how does this change the differential equation. How long would the solution to that system apply?

If the flow out is 1 gallon more than the flow in, the tank is emptying at the rate of 1 gallon per minute, so the $R_{out} = \frac{A}{500-t} \times 5 = \frac{5A}{500-t}$, which makes the differential equation: $\frac{dA}{dt} = 40 - \frac{5A}{500-t}$, which must be solved as a linear equation.

$$A' + \left(\frac{5}{500 - t}\right)A = 40$$

This equation will apply until the tank empties at $t = 500$.

2. A cup of tea is poured with a water temperature of 200-degrees and left on the counter to cool in a room temperature of 75-degrees. If it takes 5 minutes to drop to 150-degrees, what is the temperature after 10 minutes?

$$\frac{dT}{dt} = -k(T - T_0) \rightarrow \frac{dT}{T - T_0} = -kdt$$

$$\ln|T - T_0| = -kt + C \rightarrow T(t) = T_1 e^{-kt} + T_0$$

$$T(0) = 200, T_0 = 75, T(5) = 150$$

$$200 = T_1 + 75 \rightarrow T_1 = 125$$

$$150 = 125e^{-5k} + 75 \rightarrow \ln(0.6) = -5k \rightarrow k = \frac{\ln(0.6)}{-5} \approx 0.1022$$

$$T(10) = 125e^{-0.1022\times 10} + 75 = 120$$

3. Cobalt-57 is a radioactive element that takes about 272 days to decay to half its original quantity. How long has a sample of Cobalt-57 been decaying if only 3% of its original sample remains?

$$A(0) = 100\% = 1, A(272) = 50\% = 0.5$$

$$\frac{dA}{dt} = -kA \rightarrow \frac{dA}{A} = -kdt$$

$$\ln A = -kt + C \rightarrow A(t) = A_0 e^{-kt}$$

$$k = \frac{\ln(0.5)}{-272} \approx 0.002548$$

$$A(t) = e^{-0.002548t}$$

$$0.03 = e^{-0.002548t} \rightarrow \frac{\ln(0.03)}{-0.002548} = t \approx 1376\ days$$