MTH 265, Exam #1, Summer 2021 Name _____

Academic Integrity Statement

I affirm that, I, _____ (student name), do attest that I alone am completing the problems on this test without receiving unauthorized assistance. I understand that violations of academic integrity may result in sanctions, up to and including expulsion from the college.

_____(Student Signature)

_____(Student ID number)

Attach a copy of your photo ID to the online submission (there is a question drop box for it). The ID must be a photo ID. A Driver's license, School ID (NOVA or otherwise), or a work ID are acceptable as long as it contains your full name and photo.

Instructions: Show all work. Use exact answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to get full credit. Incorrect answers with no work will receive no credit. Be sure to complete all the requested elements of each problem.

1. Find $\vec{u} \times \vec{v}$ if $\vec{u} = \langle 1,2,3 \rangle$ and $\vec{v} = \langle 4,-9,0 \rangle$. (8 points)

2. Calculate the volume of the parallelepiped defined by the vectors $\langle 1,2,0 \rangle, \langle 3,4,6 \rangle, \langle 1,1,2 \rangle$. (9 points)

3. Find the equation of the plane containing the points (2,1,1), (1,2,11), and (3,2,4). (12 points)

4. Find the symmetric form of the line perpendicular to the plane $2x - 3y + z = 13$ and passes through the point (0,1,5). (8 points)

5. Rewrite $z = 2 - \sqrt{x^2 + y^2}$ in cylindrical and spherical coordinates. Simplify as much as possible. (8 points)

6. Find the limit of the following expressions at the indicated point. If the limit exists, state its value. If the limit does not exist, explain why not. You will need to test multiple paths. You may wish to use polar or spherical coordinates. (7 points each)

   a. $\lim_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^3 + y^2}$

b. $\lim_{(x,y)\to(0,0)} \dfrac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}$

c. $\lim_{(x,y,z)\to(0,0,0)} \dfrac{2x^2+yz+4xz}{x^2+y^2+z^2}$

d. $\lim_{(x,y)\to(0,0)} \dfrac{x-y}{\sqrt{x}-\sqrt{y}}$

7. Find $\vec{r}'(t)$ for vector of the vector-valued function $\vec{r}(t) = cos^2 t\hat{\imath} + sin^2 t\hat{\jmath}$. Find an expression for the length of the vector. (8 points)

8. Integrate the vector-valued function $\vec{r}(t) = \langle t, 2t^2, t^3 \rangle$ on the interval [0,2]. (7 points)

9. Find the value of the line integral $\int_C (x + 2y)dx + (3x^2 - 4y)dy$ on the curve $\vec{r}(t) = 2t\hat{\imath} - t^2\hat{\jmath}$ over the interval [0,3]. (10 points)

10. Find the value of the line integral $\int_C x^2yzds$ over the line segment connecting the points (1,2,-1) and (2,-1,3). (16 points)

11. Sketch the graph of the vector valued function $\vec{r}(t) = \sin 2t\,\hat{\imath} - 3t\hat{\jmath} + \cos 2t\,\hat{k}$. Plot at least 9 points. (10 points)

12. Sketch the vector field $\vec{F}(x, y) = xy\hat{\imath} - y\hat{\jmath}$. Plot at least 10 vectors (or more: whatever is required to determine the behaviour of the field). (12 points)

13. Find a parametric expression in two variables for the surface $z = \sqrt{x^2 + y^2}$. (7 points)