

MTH 265, Exam #2, Summer 2021

Name _____

Academic Integrity Statement

I affirm that, I, _____ (student name), do attest that I alone am completing the problems on this test without receiving unauthorized assistance. I understand that violations of academic integrity may result in sanctions, up to and including expulsion from the college.

_____(Student Signature)

_____(Student ID number)

Attach a copy of your photo ID to the online submission (there is a question drop box for it). The ID must be a photo ID. A Driver's license, School ID (NOVA or otherwise), or a work ID are acceptable as long as it contains your full name and photo.

4. Sketch six level curves for the graph $f(x, y) = xe^{-y}$. Be sure to capture all the important aspects of the graph. (10 points)
5. Find a double integral to compute the area of the region bounded by $y = 4 - x^2$ and $y = x^2 - 4$. Sketch the region. Integrate your integral to find the area. (10 points)
6. Find the volume of the solid inside the cylinder $x^2 + y^2 = 9$ above the xy -plane and below the function $\frac{1}{1+(x^2+y^2)^2}$. Set up a double or triple integral and compute the result. (13 points)

7. For $\vec{F}(x, y, z) = xz\hat{i} - \frac{y^2}{z}\hat{j} + x^2y\hat{k}$, $\vec{G}(x, y, z) = x^3\hat{i} + \ln xy\hat{j} - \frac{1}{z}\hat{k}$ and $f(x, y, z) = x^2y + e^y + yz - 11$, find each of the following. (8 points each)

a. ∇f

b. $\nabla \times \vec{G}$

c. $\nabla^2 f$

d. $\nabla \cdot \vec{F}$

8. Sketch the gradient of the function $f(x, y) = x^2 + y^2 + 2x - 6y + 15$. Clearly indicate where each component of the gradient is equal to zero. Label the general direction of the gradient in each region, and use that information to sketch at least three level curves. (10 points)

9. Integrate. (8 points each)

a. $\int_1^3 \int_x^4 \int_{x+y}^{16-x^2} xyz dz dy dx$

b. $\int_0^\pi \int_0^\pi \int_0^{\sin \varphi} \rho^2 \sin \varphi d\rho d\varphi d\theta$

c. $\int_0^{2\pi} \int_0^3 \int_0^{2-r} r^2 dz dr d\theta$

10. Find the potential function, if it exists, of the vector field $\vec{F}(x, y, z) = 2xe^{x^2}\hat{i} + (z - 3y^2)\hat{j} + \left(y + \frac{1}{z}\right)\hat{k}$.
If it does not exist, verify that it doesn't exist by showing that the curl is non-zero. (10 points)

11. Determine if the vector field $\vec{G}(x, y) = 5xy\hat{i} + \left(\frac{5}{2}x^2 + 1\right)\hat{j} + xz\hat{k}$ is conservative. (6 points)

12. Show that the mixed partial derivatives for $g(x, y) = e^{xy} + x \ln x - y^2 \sin x$ are equal. (10 points)