6/3/2021

Spherical Triple Integrals (15.9) and review for Exam #2

Examples.

Evaluate $\iiint_Q xe^{x^2+y^2+z^2}dV$, and the region is the portion of the unit ball $x^2+y^2+z^2 \leq 1$ in the first octant.

θ between 0 and $\frac{\pi}{2}$, and ϕ is between 0 and $\frac{\pi}{2}$ is the first octant. The sphere is just ρ between 0 and 1.

$$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^1 \rho\cos\theta\sin\phi\, e^{\rho^2}\rho^2\sin\phi\, d\rho d\phi d\theta$$

Example.
Find the volume of part of the ball $\rho \leq 2$, that lies between the cones $\phi = \frac{\pi}{6}$, and $\phi = \frac{\pi}{3}$.

$$\int_0^{2\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\int_0^2 \rho^2\sin\phi\, d\rho d\phi d\theta$$

Example.
Find the volume of the region that lies within the sphere $x^2+y^2+z^2=4$, above the xy-plane, and below the cone, $z=\sqrt{x^2+y^2}$.

$$\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^2 \rho^2\sin\phi\, d\rho d\phi d\theta$$

Example.

$$\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xydzdydx$$

$$z=\sqrt{x^2+y^2}, z=\sqrt{2-x^2-y^2}$$

$$\sqrt{x^2+y^2}=\sqrt{2-x^2-y^2}$$

$$2x^2+2y^2=2 \rightarrow x^2+y^2=1$$

$$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{2}} \rho\cos\theta\sin\phi\,\rho\sin\theta\sin\phi\,\rho^2\sin\phi\, d\rho d\phi d\theta$$

Example.

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}}(x^2+y^2+z^2)^{\frac{3}{2}}dzdydx$$

$$z = 2+\sqrt{4-x^2-y^2}$$
$$z = 2-\sqrt{4-x^2-y^2}$$
$$z-2 = \pm\sqrt{4-x^2-y^2}$$
$$(z-2)^2 = 4-x^2-y^2$$
$$x^2+y^2+(z-2)^2 = 4$$
$$x^2+y^2+z^2-4z+4 = 4$$
$$x^2+y^2+z^2 = 4z$$
$$\rho^2 = 4\rho\cos\phi$$
$$\rho = 4\cos\phi$$

$$\int_{0}^{2\pi}\int_{0}^{\frac{\pi}{2}}\int_{0}^{4\cos\phi}\rho^3\rho^2\sin\phi\, d\rho d\phi d\theta$$

In spherical, a torus $\rho = \sin\phi$.

End of material for Exam #2.