MTH 265, Quiz #10, Summer 2021 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the curvature of the vector-valued function $\vec{r}(t) = t^2\hat{\imath} + t\sin\pi t\,\hat{\jmath} + 2t\hat{k}$, at the point (1,0,2). Use the formula $\kappa = \frac{\|\vec{r}' \times \vec{r}''\|}{\|\vec{r}'\|^3}$.

$t = 1$

$r'(t) = 2t\hat{\imath} + (\sin\pi t + t\pi\cos\pi t)\hat{\jmath} + 2\hat{k}$

$r''(t) = 2\hat{\imath} + (\pi\cos\pi t + \pi\cos\pi t - t\pi^2\sin\pi t)\hat{\jmath} + 0\hat{k} =$

$\langle 2,\ 2\pi\cos\pi t - t\pi^2\sin\pi t\rangle$

$r'(t) = \langle 2, -\pi, 2\rangle \quad r''(1) = \langle 2, -2\pi, 0\rangle$

$\|r' \times r''\| = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -\pi & 2 \\ 2 & -2\pi & 0 \end{vmatrix} = \|(4\pi)\hat{\imath} - (-4)\hat{\jmath} + (-4\pi + 2\pi)\hat{k}\| =$

$\|\langle 4\pi, 4, -2\pi\rangle\| =$

$\sqrt{16\pi^2 + 16 + 4\pi^2} = \sqrt{20\pi^2 + 16} = 2\sqrt{5\pi^2 + 4}$

$\|r'(1)\| = \sqrt{4 + \pi^2 + 4} = \sqrt{8 + \pi^2}$

$$\kappa(1) = \frac{2\sqrt{5\pi^2+4}}{(8+\pi^2)^{3/2}} \approx 0.19338$$

2. Find the surface area for the function $f(x, y) = 4 - x^2 - y^2$ above the plane.

$f_x = -2x \quad f_y = -2y \quad \iint_R \sqrt{1 + 4x^2 + 4y^2}\, dA$

$x^2 + y^2 = 4$

$r = 2$

$\int_0^{2\pi}\int_0^2 \sqrt{1 + 4r^2}\, r\,dr\,d\theta \qquad u = 1 + 4r^2$

$\frac{du}{8} = \cancel{8} r\,dr$

$\int \frac{1}{8} u^{1/2}\,du = \frac{1}{\cancel{8}4} \cdot \frac{\cancel{2}}{3} u^{3/2} = \frac{1}{12}(1 + 4r^2)^{3/2}\Big|_0^2 = \frac{1}{12}\left[17^{3/2} - 1\right]$

$\int_0^{2\pi} \frac{1}{12}(17^{3/2} - 1)\,d\theta = \frac{\pi}{6}(17^{3/2} - 1)$