MTH 265, Quiz #13, Summer 2021   Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the critical points of the function $f(x,y) = x^2 - 3xy + 2y^2 + 4y$ and characterize each as a maximum, minimum or saddle point.

$f_x = 2x - 3y = 0 \quad 2x = 3y \Rightarrow x = \frac{3}{2}y$

$f_y = -3x + 4y + 4 = 0 \Rightarrow -3\left(\frac{3}{2}y\right) + 4y + 4 = 0 \Rightarrow -y/2 = -4$

$\Rightarrow y = 8,\ x = 12$

$(12, 8)$

$f_{xx} = 2$

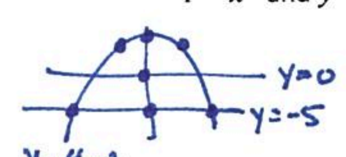
$f_{yy} = 4$

$f_{xy} = -3$

$(2)(4) - (-3)^2 = -1$

Saddle point

2. Find the absolute extrema for the function $f(x,y) = x^2 - 2y^2$ on the region bounded by $y = 4 - x^2$ and $y = -5$.



① $f_x = 2x = 0 \Rightarrow x = 0 \quad f_y = -4y = 0 \Rightarrow y = 0 \quad f(0,0) = 0$

$f(0,4) = -32$

② $f_1(x) = x^2 - 2(4 - x^2) = x^2 - 2(16 - 8x^2 + x^4) =$ $f(\pm 3/2, 7/4) =$

$x^2 - 32 + 8x^2 - 2x^4 = -2x^4 + 9x^2 - 32 \quad -31/8$

$f'(x) = -8x^3 + 18x = 0 = -2x(4x^2 - 9) \quad x = 0,\ x^2 = 9/4,\ x = \pm\frac{3}{2}$

$y = 4 - (\pm 3/2)^2 = 4 - 9/4 = 7/4$

$f(x) = x^2 - 50 \quad f'(x) = 2x \to x = 0 \quad (0,-5) \quad f(0,-5) = -50$

③ corners.

$y = -5 = 4 - x^2$

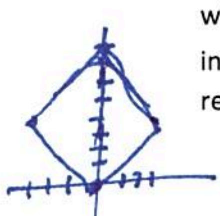
$x^2 = 9 \quad x = \pm 3$

$(-3,5), (-3,-5) \quad f(\pm 3, -5) = -41$

ABS max $f(0,0) = 0$

ABS min $f(0,-5) = -50$

3. Find an appropriate change of variables for the region bounded by the sides of the rectangle with vertices (0,0), (3,4), (0,8), and (-3,4). Calculate the value of the Jacobian, and use that information to find the value of the integral $\int_A \int (3x - 4y)^2 e^{3x+4y}\, dA$ over the indicated rectangle.



$m = -\frac{4}{3} \Rightarrow y = -\frac{4}{3}x \Rightarrow y - 4 = -\frac{4}{3}(x - 3) \Rightarrow y = -\frac{4}{3}x + 8$

$4x + 3y = 0 \quad \$ \quad 4x + 3y = 24 \Rightarrow u = 4x + 3y \quad [0, 24]$

$m = \frac{8-4}{0+3} = \frac{4}{3} \Rightarrow y = \frac{4}{3}x + 8 \quad m = \frac{4-0}{0+3} = \frac{4}{3} \Rightarrow y = \frac{4}{3}x \quad 3y - 4x = 24$

$3y - 4x = 0 \quad [0, 24]$

$\left.\begin{array}{l} -4x + 3y = v \\ 4x + 3y = u \end{array}\right\rangle \; 6y = v + u \quad y = \frac{1}{6}(v + u)$

$\left.\begin{array}{l} 4x - 3y = -v \\ 4x + 3y = u \end{array}\right\rangle \; 8x = u - v \Rightarrow x = \frac{1}{8}(u - v)$

$J = \begin{vmatrix} 1/8 & -1/8 \\ 1/6 & 1/6 \end{vmatrix} = \frac{1}{48} + \frac{1}{48} = \frac{1}{24}$

$p = u^2 \quad dg = e^u \qquad p = 2u \quad dg = e^u du$

$dp = 2u \quad g = e^u \qquad dp = 2du \quad g = e^u$

$\frac{1}{24}\int_0^{24}\int_0^{24} u^2 e^u\, du\, dv = \int_0^{24} u^2 e^u\, du = u^2 e^u - \int 2u e^u\, du =$

$u^2 e^u - 2ue^u + 2e^u \Big|_0^{24} = e^{24}[530] - 2$