MTH 265, Quiz #5, Summer 2021 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the three first partial derivatives for $f(x, y, z) = e^{-xy}\sin(yz)$.

$$f_x = -ye^{-xy}\sin(yz)$$

$$f_y = -xe^{-xy}\sin(yz) + e^{-xy} z\cos(yz)$$

$$f_z = ye^{-xy}\cos(yz)$$

2. Find the total differential for $z = ye^x$ at the point (0,1) and use this information to approximate the function at (0.1, 0.95).

$$dz = ye^x dx + e^x dy$$

$$dz(0,1) = 1\,dx + 1\,dy$$

$$dx = 0.1$$

$$dy = -0.05$$

$$dz \approx 1(.1) + (1)(-.05)$$

$$= 0.05$$

$$z(0,1) + 0.05 = 1e^0 + 0.05 = 1.05$$

3. Integrate. $\int_1^4 \int_1^{\sqrt{x}} 2ye^{-x}dydx$

$$\int_1^4 y^2 e^{-x} \Big|_1^{\sqrt{x}} dx = \int_1^4 (x-1)e^{-x}dx$$

$u = x-1$

$du = dx$

$dv = e^{-x}dx$

$v = -e^{-x}$

$$-(x-1)e^{-x} + \int e^{-x}dx = -(x-1)e^{-x} - e^{-x}\Big|_1^4$$

$$-3e^{-4} - e^{-4} - (0 - e^{-1}) = -4e^{-4} + e^{-1}$$

$$= \frac{1}{e} - \frac{4}{e^4}$$

4. Find the gradient of the function $f(x,y) = xy(1-x^2-y^2)$.

$$= xy - x^3y - xy^3$$

$$\nabla f = \langle y - 3x^2 - y^3,\ x - x^3 - 3xy^2 \rangle$$

5. Find the gradient of the function $f(x,y) = \sin(xy^2)$. Sketch key features of the gradient and the general direction of the gradient in each region. Use this information to sketch some level curves of the function.

$$\nabla f = \langle \cos(xy^2)y^2,\ \cos(xy^2)\cdot 2xy \rangle$$

$\cos(xy^2)y^2 = 0 \rightarrow \cos(xy^2) = 0$ or $y^2 = 0 \rightarrow y = 0$

$\cos(xy^2)(2xy) = 0 \rightarrow \cos(xy^2) = 0$ or $2xy = 0 \rightarrow x = 0,\ y = 0$

$xy^2 = \pi/2,\ 3\pi/2 \Rightarrow y = \pm\sqrt{\pi/2x},\ \pm\sqrt{3\pi/2x}$ (etc)

$(-1,1) \rightarrow \langle +\ \ - \rangle \searrow$

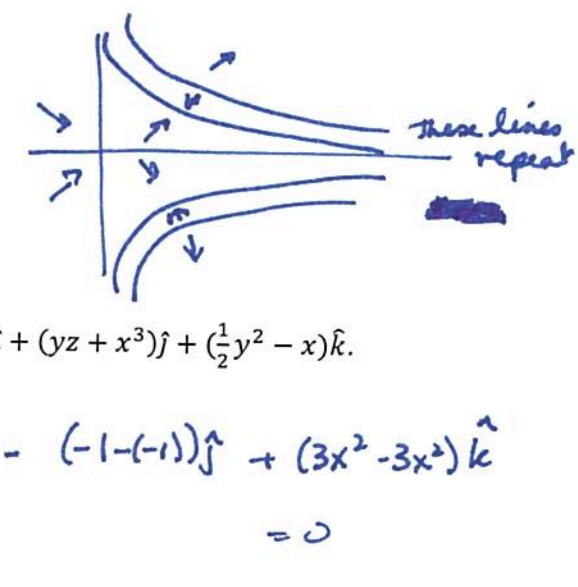
$(-1,-1) \rightarrow \langle +, + \rangle \nearrow$ (0,0) saddle point

$(1,1) \rightarrow \langle +, + \rangle \nearrow$

$(1,-1) \rightarrow \langle +, - \rangle \searrow$

$y = \pm\sqrt{\pi/2x}$ peak of wave

$y = \pm\sqrt{3\pi/2x}$ trough of wave



6. Find $\nabla \times F$ for the vector field $F(x,y,z) = (3x^2y - z)\hat{\imath} + (yz + x^3)\hat{\jmath} + (\frac{1}{2}y^2 - x)\hat{k}$.

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2y - z & yz + x^3 & \frac{1}{2}y^2 - x \end{vmatrix} = (y-y)\hat{\imath} - (-1-(-1))\hat{\jmath} + (3x^2 - 3x^2)\hat{k}$$

$$= 0$$

$$\nabla \times F = \vec{0}$$