MTH 265, Quiz #8, Summer 2021 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the unit tangent vector, the unit normal vector and the binormal vector for the helix $\vec{r}(t) = 5\cos 2t\,\hat{\imath} + 5\sin 2t\,\hat{\jmath} + 3t\hat{k}$.

$$r'(t) = -10\sin 2t\,\hat{\imath} + 10\cos 2t\,\hat{\jmath} + 3\hat{k}$$

$$\|r'(t)\| = \sqrt{100\sin^2 2t + 100\cos^2 2t + 9} = \sqrt{100+9} = \sqrt{109}$$

$$T(t) = \frac{-10}{\sqrt{109}}\sin 2t\,\hat{\imath} + \frac{10}{\sqrt{109}}\cos 2t\,\hat{\jmath} + \frac{3}{\sqrt{109}}\hat{k}$$

$$T'(t) = \frac{-20}{\sqrt{109}}\cos 2t\,\hat{\imath} - \frac{20}{\sqrt{109}}\sin 2t\,\hat{\jmath} + 0\hat{k}$$

$$\|T'(t)\| = \left(\frac{20}{\sqrt{109}}\right)$$

$$N(t) = -\cos 2t\,\hat{\imath} - \sin 2t\,\hat{\jmath}$$

$$B(t) = T(t) \times N(t) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{-10}{\sqrt{109}}\sin 2t & \frac{10}{\sqrt{109}}\cos 2t & \frac{3}{\sqrt{109}} \\ -\cos 2t & -\sin 2t & 0 \end{vmatrix} =$$

$$\left(0 + \frac{3}{\sqrt{109}}\sin 2t\right)\hat{\imath} - \left(0 + \frac{3}{\sqrt{109}}\cos 2t\right)\hat{\jmath} + \left(\frac{10}{\sqrt{109}}\sin^2 2t + \frac{10}{\sqrt{109}}\cos^2 2t\right)\hat{k} = \left\langle \frac{3}{\sqrt{109}}\sin 2t, \frac{3}{\sqrt{109}}\cos 2t, \frac{10}{\sqrt{109}} \right\rangle$$

2. Find the unit tangent vector for the vector-valued function $\vec{r}(t) = t\cos t\,\hat{\imath} + 3e^t \sin t\,\hat{\jmath} + 2te^t\hat{k}$. In what direction is it pointing when $t = \pi$.

$$r'(t) = (\cos t - t\sin t)\hat{\imath} + (3e^t\sin t + 3e^t\cos t)\hat{\jmath} + (2e^t + 2te^{-t})\hat{k}$$

$$r'(\pi) = [(-1) - \pi(0)]\hat{\imath} + (3e^{\pi}(0) - 3e^{\pi}((-1)))\hat{\jmath} + (2e^{\pi} + 2\pi e^{\pi})\hat{k} =$$

$$\langle -1, -3e^{\pi}, 2e^{\pi}(1+\pi) \rangle$$

$$\|r'(\pi)\| = \sqrt{(-1)^2 + (3e^{\pi})^2 + [(2e^{\pi})(1+\pi)]^2} = \sqrt{1 + 9e^{2\pi} + 4e^{2\pi}(1 + 2\pi + \pi^2)} =$$

$$\sqrt{1 + 13e^{2\pi} + 8\pi e^{2\pi} + 4\pi^2 e^{2\pi}}$$

$$T(\pi) = \frac{r'(\pi)}{\|r'(\pi)\|} = \frac{\langle -1, -3e^{\pi}, (2e^{\pi})(1+\pi) \rangle}{\sqrt{1 + 13e^{2\pi} + 8\pi e^{2\pi} + 4\pi^2 e^{2\pi}}}$$