MTH 265, Quiz #9, Summer 2021 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the directional derivative of the function $f(x, y) = x \sin y - e^{xy}$ at the point $\left(1, \frac{\pi}{2}\right)$, in the direction of $\vec{u} = 5\hat{\imath} - 8\hat{\jmath}$.

$\|u\| = \sqrt{25+64} = \sqrt{89}$

$\hat{u} = \left\langle \frac{5}{\sqrt{89}}, \frac{-8}{\sqrt{89}} \right\rangle$

$\nabla f = \langle \sin y - ye^{xy}, x\cos y - xe^{xy} \rangle$

$\nabla f(1, \pi/2) = \langle 1 - \pi/2\, e^{\pi/2}, -e^{\pi/2} \rangle$

$\nabla f \cdot \hat{u} = \langle 1 - \pi/2\, e^{\pi/2}, -e^{\pi/2} \rangle \cdot \left\langle \frac{5}{\sqrt{89}}, \frac{-8}{\sqrt{89}} \right\rangle$

$\frac{5\left(\frac{2}{2}1 - \pi/2\, e^{\pi/2}\right)}{\sqrt{89}} + \frac{2 \cdot 8e^{\pi/2}}{2 \cdot \sqrt{89}} = \frac{10 + (16 - 5\pi)e^{\pi/2}}{2\sqrt{89}} \approx 0.6044$

2. Find the equation of the tangent plane for $f(x, y) = x^2y - xy^3$, at the point (1,-2).

$F(x,y,z) = x^2y - xy^3 - z$

$\nabla F = \langle 2xy - y^3, x^2 - 3xy^2, -1 \rangle$

$\nabla F = \langle -4 - (-8), 1 - 12, -1 \rangle = \langle 4, -11, -1 \rangle$

$z_0 = -2 - (-8) = 6$

$4(x-1) - 11(y+2) - (z-6) = 0$

3. Find the equation of the tangent plane for the parametric surface given by $\vec{r}(u, v) = u\cos v\,\hat{\imath} + u\sin v\,\hat{\jmath} + uv^3\hat{k}$ at the point $(-3, 0, -3\pi^3)$.

$\vec{r}_u = \cos v\,\hat{\imath} + \sin v\,\hat{\jmath} + v^3\hat{k}$

$r_v = -u\sin v\,\hat{\imath} + u\cos v\,\hat{\jmath} + 3uv^2\hat{k}$

$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \cos v & \sin v & v^3 \\ -u\sin v & u\cos v & 3uv^2 \end{vmatrix} = (\cancel{3uv^2\sin v}^{\,0} - v^3u\cos v)\hat{\imath} - (3uv^2\cos v + \cancel{uv^3\sin v}^{\,0})\hat{\jmath} + (u\cos^2 v + u\sin^2 v)\hat{k}$

$= u\hat{k}$

$uv^3 = -3\pi^3$

$u\sin v = 0$

$v = \pi, -\pi, 0$, etc.

$u\cos v = -3$

$\cos\pi = -1$

$u = 3$

$\cos(-\pi) = -1$

$u = 3$

$v = -\pi$

$u = 3$

$= (-\pi)^3(3)(-1)\hat{\imath} - 3(3)(-\pi)^2(-1)\hat{\jmath} + 3\hat{k} = \langle 3\pi^3, 9\pi^2, 3 \rangle \rightarrow 3\pi^3(x+3) + 9\pi^2(y) + 3(z - 3\pi^3) = 0$