MTH 264, Homework #10, Summer 2022   Name ________________________________

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

Note: In this homework, some of the integrals can be calculated with various calculus and algebraic techniques learned in class. Find exact values for these integrals. If they cannot be calculated by techniques learned in this course, then you may calculate the integrals numerically: use $n = 6$ if you use Simpson's Rule, or $n = 10$ if you use the Trapezoidal rule. You may confirm your result with a numerical integration program in your calculator or MatLab, but you must show work by hand for all problems.

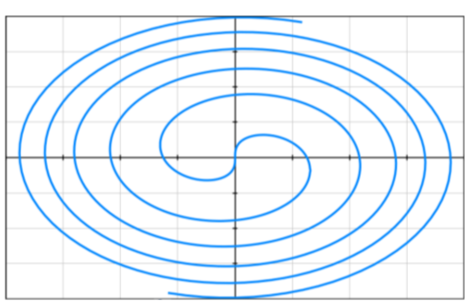
1. For each of the parametric curves, sketch the graph and indicate its orientation. Write the non-parametric form of the equation.
   a. $x = t + 1, y = t^2$
   b. $x = t - 1, y = \dfrac{t}{t-1}$
   c. $x = \tan^2\theta, y = \sec^2\theta$ [Hint: Recall $1 + \tan^2\theta = \sec^2\theta$]
   d. $x = \frac{1}{2}\cos\theta\,, y = 2\sin\theta$ [Hint: Recall $\sin^2\theta + \cos^2\theta = 1$]
   e. $x = \sinh t\,, y = \cosh t$
   f. $x = 3 - 2t, y = 2 + 3t$
   g. $x = e^{-t} y = e^{2t} - 1$
   h. $x = \cos^2 t\,, y = 1 - \sin t$

2. Find parametric equations for the graphs described.
   a. Line passing through $(1,4)$ and $(5, -2)$
   b. Circle centered at $(-3,1)$ and radius 3

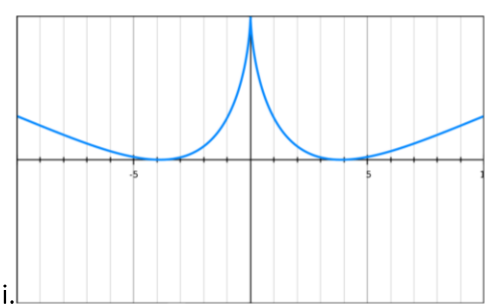
3. For each of the parametric curves in problem #2, find all points of vertical and horizontal tangency to the curve. Then determine the intervals on which the curve in concave upward/downward. Use the first and second derivatives in parametric form.

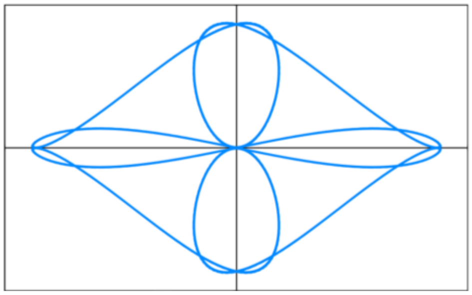
4. Match the parametric equation to its graph.
   a. $x = t\sin(t^2)\,, y = t\cos(t^2)$
   b. $x = t^3, y = \cos^2 t$
   c. $x = e^{-t} + e^t - 10, y = \sqrt{|t^2 - 4|}\sin(t)$
   d. $x = \sin(t)\,cos(3t)\,, y = \sin^2(3t)\cos t$

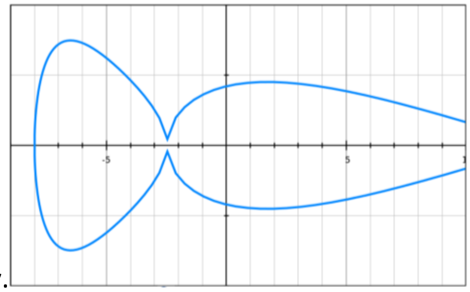
i.



iii.



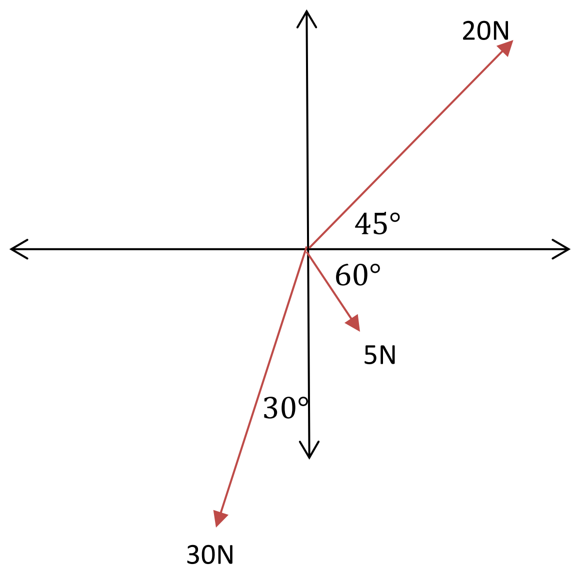
ii. 

iv. 

5. Convert the plane curve (2-D) into a vector-valued function. There is not only one correct answer, but use the simplest method possible that captures the whole curve.
   a. $y = 4 - x$
   b. $y = 4 - x^2$
   c. $x^2 + y^2 = 25$

6. Find the component vector given the magnitude and the angle the vector makes with the positive x-axis. Use the formula $r = \|\vec{v}\|, \vec{v} = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath}$. Round to two decimal places.
   a. $\|\vec{v}\| = 5, \theta = 120°$
   b. $\|\vec{v}\| = 8, \theta = -3.5\,rad$

7. For the vector $20\hat{\imath} + 20\hat{\jmath}$, find the magnitude and direction of the vector.



8. Find the total force and direction resulting from adding the forces shown above.

9. Find the dot product of the following pairs of vectors.
   a. $\vec{u} = \langle 1, 3\rangle, \vec{v} = \langle 5, -2\rangle$
   b. $\vec{u} = \langle 9, -4\rangle, \vec{v} = \langle 3, -2\rangle$

10. Find the projection of $\langle 1, -1\rangle$ onto the vector $\langle 3, 4\rangle$.

11. Find a vector orthogonal to $\langle 4,2\rangle$.

12. Find the work done by the force $\vec{F} = 8\hat{\imath} - 6\hat{\jmath}$ that moves an object from the point $(0,10)$ to $(6,12)$ along a straight line if the distance is measured in meters and force in newtons.

13. Find the domain of the vector-valued function. Find an expression for $\left\|\overrightarrow{r(t)}\right\|$. Sketch the curve, and note its orientation.
   a. $\overrightarrow{r(t)} = \sqrt{4-t^2}\,\vec{i} + t^2\vec{j} - 6t\vec{k}$
   b. $\overrightarrow{r(t)} = 3\cos t\,\vec{i} + 2\sin t\,\vec{j} + t^2\vec{k}$
   d. $\overrightarrow{r(t)} = (\ln t - 1)\vec{i} + t\vec{j}$
   e. $\overrightarrow{r(t)} = \sqrt[3]{t}\,\vec{i} + \dfrac{1}{t+1}\vec{j} + (t+2)\vec{k}$

c. $\overrightarrow{r(t)} = (1-t)\vec{i} + \sqrt{t}\vec{k}$

14. Find the arclength of the curve on the given interval. You may need to integrate numerically.

a. $x = t^2, y = 2t \quad [0,2]$

b. $x = a\cos^3\theta, y = a\sin^3\theta \quad [0,2\pi]$

c. $x = e^t + e^{-t}, y = 5 - 2t, [0,3]$

d. $x = 2 + 3t, y = \cosh 3t, [0,1]$

e. $x = \arcsin t, y = \ln\sqrt{1-t^2} \quad \left[0, \frac{1}{2}\right]$

f. $x = 3\cos t - \cos 3t, y = 3\sin t - \sin 3t, [0,\pi]$

g. $x = \cos t + \ln\left[\tan\left(\frac{1}{2}t\right)\right], y = \sin t, \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

15. The Gateway Arch in St. Louis, Missouri is modeled by $y = 693.8597 - 68.7672\cosh(0.0100333x)$ on the interval [-299.2239,299.2239]. Find the length of the arch.