MTH 264, Homework #4, Summer 2022 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Use a table of integrals to integrate each of the following. Make any substitutions necessary. State the formula used (from the attached table).
   a. $\int_0^{\pi/2} \cos 5x \cos 2x \, dx$
   b. $\int_0^2 x^2\sqrt{4-x^2}dx$
   c. $\int \frac{\tan^3\left(\frac{1}{z}\right)}{z^2} dz$
   d. $\int \frac{\sin 2\theta}{\sqrt{5-sin\theta}} d\theta$
   e. $\int \sin^2 x \cos x \ln(\sin x) \, dx$
   f. $\int \sqrt{e^{2t}-1}dt$
   g. $\int e^t \sin(\alpha t - 3) \, dt$
   h. $\int \frac{5t}{(2t+1)^7} dt$
   i. $\int \sqrt{\frac{x-7}{x+7}} dx$
   j. $\int \frac{\sqrt{16+\frac{1}{x^2}}}{x^3} dx$
   k. $\int \sin^2 2\phi \cos 3\phi \, d\phi$
   l. $\int x^4 \arctan x \, dx$
   m. $\int q^2 \ln(q^6+9) \, dq$
   n. $\int \log_3(4x-7) \, dx$
   o. $\int ye^{3y} \sin 3y \, dy$
   p. $\int_1^2 \sqrt{4x^2-3} \, dx$
   q. $\int \frac{\cos x}{\sin^2 x - 9} dx$
   r. $\int x \sin x^2 \cos(3x^2) \, dx$
   s. $\int \sin^6 2x \, dx$
   t. $\int \sec^5 x \, dx$
   u. $\int \frac{1}{\sqrt{1+\sqrt[3]{x}}} dx$
   v. $\int \frac{1}{2x^2-3x+5} dx$
   w. $\int \frac{\sqrt{e^x+1}}{e^{3x}} dx$
   x. $\int \sin\theta \cos\theta \sqrt{\cos^2\theta + 5\cos\theta + 4}d\theta$
   y. $\int \frac{e^{3x}}{(e^{2x}-8)^{\frac{3}{2}}} dx$
   z. $\int \frac{1}{1+\tan p} dp$
   aa. $\int e^{2x} \sinh(\frac{1}{2}e^{2x}) \sin(3e^{2x})dx$
   bb. $\int \ln^3(x-1) \, dx$
   cc. $\int \frac{11}{2+3e^{t/10}} dt$
   dd. $\int e^{-x} \cosh 2x \, dx$

2. Use the fact that $\sin t = \frac{e^{it}-e^{-it}}{2i}$ to evaluate $\int e^{2\theta} \sin 3\theta d\theta$ without "looping". Be sure to convert back to an expression involving real values only. Confirm that both methods produce the same result.

3. Integrate using any appropriate method.
   a. $\int \text{sech}^3 x \tanh x dx$
   b. $\int t \cot t \csc t dt$
   c. $\int x \,\text{arcsec}\, x dx$
   j. $\int x\sqrt{4-x}dx$
   k. $\int 4 \arccos x dx$
   l. $\int x^2(x-2)^{3/2} dx$

d. $\int_0^{\pi/3} \tan^2 x dx$

e. $\int \cos^4 x dx$

f. $\int \frac{\cot^3 \theta}{\csc \theta} d\theta$

g. $\int e^{2x}\sqrt{1+e^{2x}} dx$

h. $\int \frac{4x^2}{x^3+x^2-x-1} dx$

i. $\int \frac{\sec^2 x}{\tan x(\tan x+1)} dx$

m. $\int \sin^5 x \cos^2 x dx$

n. $\int \sin x \tan^2 x dx$

o. $\int \frac{\sqrt{x^2+16}}{x} dx$

p. $\int_3^6 \frac{\sqrt{x^2-9}}{x^2} dx$

q. $\int_0^1 \frac{x^2-x}{x^2+x+1} dx$

r. $\int \frac{1}{t\left[1+(\ln t)^2\right]} dt$

4. Find the limit.

a. $\lim_{x\to 0} \frac{\arcsin x}{x}$

b. $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{1/x}$

c. $\lim_{x\to\infty} \frac{x^2}{\sqrt{x^2+1}}$

d. $\lim_{x\to 0^+} x^3 \cot x$

5. Determine if the integral converges or diverges. If it converges, give the value of the area under the curve.

a. $\int_0^1 \frac{1}{\sqrt[3]{x}} dx$

b. $\int_{-\infty}^1 e^x dx$

c. $\int_0^1 \frac{e^{\frac{1}{x}}}{x^2} dx$

d. $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$

e. $\int_2^\infty \frac{1}{\sqrt{x-1}} dx$

f. $\int_0^9 \frac{1}{\sqrt[3]{x-1}} dx$

g. $\int_2^\infty \frac{1}{\sqrt[4]{1+x}} dx$

h. $\int_1^\infty \frac{\ln x}{x} dx$

6. Find all the values for p for which $\int_1^\infty \frac{1}{x^p} dx$ converges.

7. For each integral, determine the integration strategy, algebraic identity, substitution and/or differentiation rule needed to evaluate each integral. You don't need to fully evaluate the integral, but if more than one rule or strategy needs to be employed in succession, you need to go far enough to state them all. Be explicit about substitutions or identities used.

a. $\int \frac{\sin x+\sec x}{\tan x} dx$

b. $\int \frac{dx}{(1-x^2)^{\frac{3}{2}}}$

j. $\int \frac{x}{x^4+x^2+1} dx$

k. $\int \frac{\arctan x}{x^2} dx$

c. $\int x^5 e^{-x^3}\, dx$

d. $\int_0^1 (3x+1)^{\sqrt{2}}\, dx$

e. $\int_0^1 x\sqrt{2-\sqrt{1-x^2}}\, dx$

f. $\int \arctan\sqrt{x}\, dx$

g. $\int \sqrt{1+e^x}\, dx$

h. $\int \ln\left(x+\sqrt{x^2-1}\right) dx$

i. $\int \frac{\sec\theta\tan\theta}{\sec^2\theta-\sec\theta}\, d\theta$

l. $\int \frac{\sqrt{x}}{1+x^3}\, dx$

m. $\int \frac{\ln x}{x\sqrt{1+\ln^2 x}}\, dx$

n. $\int_0^4 \frac{6z+5}{2z+1}\, dz$

o. $\int e^2\, dx$

p. $\int \cos 2x \cos 6x\, dx$

q. $\int \frac{dx}{1+e^x}$

8. Explain why each integral is improper. Note all problem areas. Rewrite the integral as a limit. You don't need to determine convergence.

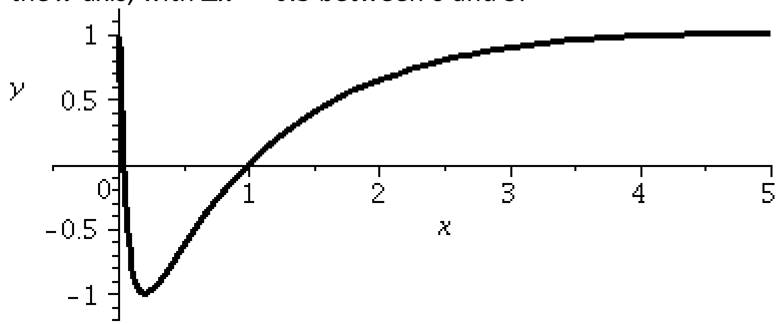
a. $\int_0^9 \frac{x}{x-1}\, dx$

b. $\int_0^{\pi} \tan x\, dx$

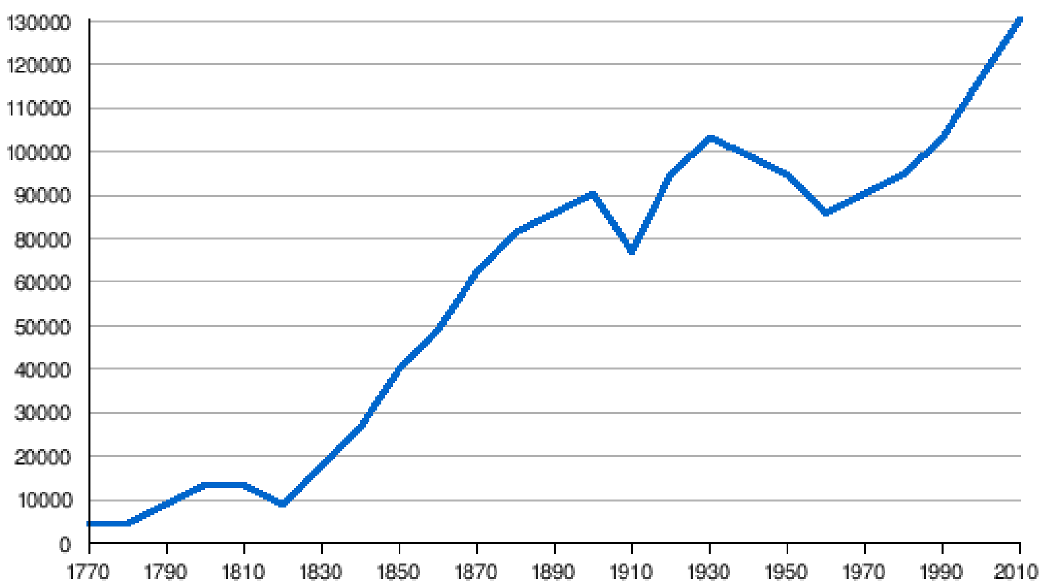
c. $\int_0^{\infty} \frac{1}{1+x^3}\, dx$

d. $\int_{-1}^{1} \frac{dx}{x^2-x-2}$

9. Using the graph below and the Trapezoidal Rule, estimate the area bounded by the graph and the x-axis, with $\Delta x = 0.5$ between 0 and 5.



10. Using the graph below and Simpson's Rule, estimate the area under the graph between 1770 and 2010. Use units of ten-thousands for the y-values and $\Delta t = 10$ years.



11. Numerically integrate each of the following functions using:
    a. The Trapezoidal Rule
    b. Simpson's Rule
    c. Compare your results to the true value from the Fundamental Theorem of Calculus (for (i) only).
        i. $\int_0^8 \sqrt[3]{x}dx, n = 8$
        ii. $\int_1^4 \sqrt{\ln x}\, dx, n = 6$
        iii. $\int_{-1}^1 e^{e^x} dx, n = 10$

12. Use the Error formulas to calculate the number of partitions needs to calculate each integral to within 0.000001 for:
    a. The Trapezoidal Rule
    b. Simpson's Rule.
        i. $\int_1^3 \frac{1}{\sqrt{x}} dx$
        ii. $\int_{-1}^1 \sqrt{4 - x^3} dx$
        iii. $\int_1^2 e^{1/x} dx$