MTH 264, Homework #9, Summer 2022 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Verify that each function is a solution to given differential equation.
   a. $y'' - y = 0, y_1(t) = e^t, y_2(t) = \cosh(t)$
   b. $y^{IV} + 4y''' + 3y = t, y_1(t) = \frac{t}{3}, y_2(t) = \frac{t}{3} + e^{-t}$
   c. $ty' - y = t^2, y_1(t) = 3t + t^2$

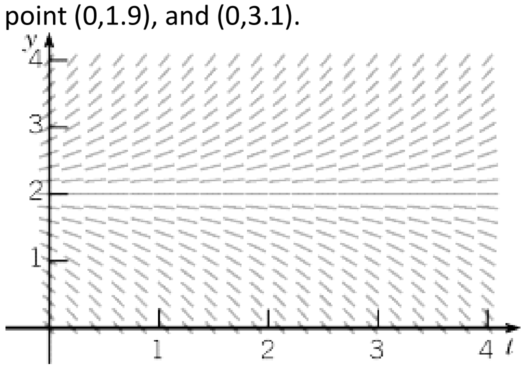
2. Find the values of r for which the differential equation has a solution of the given form.
   a. $y'' + y' - 6y = 0, y = e^{rt}$
   b. $t^2y'' + 4ty' + 2y = 0, y = t^r$

3. Graph the direction field for each autonomous equation by hand and comment on the stability of each equilibrium.
   a. $y' = 1 + 2y$
   b. $y' = -y(5 - y)$
   c. $y' = y(y - 2)^2$

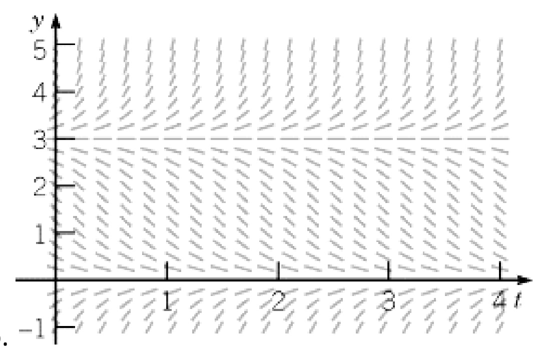
4. Use a direction field graphing program to graph the following direction fields.
   a. $y' = e^{-t} + y$
   b. $y' = \frac{1}{6}y^3 - y - \frac{1}{3}t^2$
   c. $y' = t + 2y$

5. For the direction fields below, state the differential equation that produces the graph (assume the equilibria are integer values). Plot the path a particle would take in the field if it started at the point (0,1.9), and (0,3.1).

a.



b.



6. Determine whether the equation $\frac{d^3y}{dt^3} + t\frac{dy}{dt} + (cos^2 t)y = t^3$ is linear or nonlinear. Explain why or why not? What is the order of the equation? Is it ordinary or partial?

7. Solve the following differential equations by separation of variables. If an initial value is provided, be sure to find all unknown constants. If the solution is not valid for all values of (t,y) state the intervals where it is valid.
   a. $y' = \frac{x^2}{y(1+x^3)^4}$
   b. $y' = \frac{1-2x}{y}, y(1) = -2$
   c. $y' + y^2 \sin x = 0$
   d. $xy' = (1 - y^2)^{\frac{1}{2}}$
   e. $\sin 2x\, dx + \cos 3y\, dy = 0, y\left(\frac{\pi}{2}\right) = \frac{\pi}{3}$