

6/10/2023

Preparing for fractions:

Least common multiples

Greatest common factor

Factoring: primes, composite, relatively prime

Divisibility rules

Multiply/divide fractions

Add/subtract fractions

Prime number is a number that is divisible only by 1 and itself.

2, 3, 5, 7, 11, 13, 17, 19, 23, etc.

Numbers that are not prime are called composite numbers (other than 1). Divisible by more than one prime.

Numbers that have no factors in common are considered relatively prime.

6 and 25

Divisibility rules:

Division by 2: Any number that is divisible by 2 is even (the last digit is any of 0, 2, 4, 6, 8)

Division by 3: Any number whose digits add up to a number divisible by 3 is divisible by 3

$820,572 : 8+2+0+5+7+2=10+5+9=15+9=24$ . 24 is divisible by 3:  $24/3 = 8$ .

This original number 820,572 also divides evenly by 3:  $820,572/3=273,524$

71:  $7+1=8$ , 8 is not divisible by 3,  $71/3 = 23.666666...$

Division by 4: any number where the last two digits are divisible by 4 is divisible by 4.

820,572 is divisible by 4 because 72 is divisible by 4.

Division by 5: any number whose last digit is 5 or 0 is divisible by 5.

Division by 6: any number that is both divisible by 2 (even) and divisible by 3 is divisible by 6.

820,572: the final digit is 2, and the digits add up to 24 which is divisible by 3, this number is also divisible by 6.

Division by 7: hard, and so use your calculator

Division by 8: any number where the last three digits are divisible by 8 is divisible by 8.

Division by 9: any number whose digits add up to a number that is itself divisible by 9 is divisible by 9.

820,575:  $8+2+0+5+7+5=10+12+5=27$ . 27 is divisible by 9, and so 820,575 is divisible by 9.

Division by 10: any number whose last digit is 0 is divisible by 10

Other divisibility rules are hard.

Factoring:

Start with 72.

Canonical way to factor 72 (the way it shows up in our multiplication tables is  $8 \times 9$ )

72 is even, so it does divide by 2:  $2 \times 36$

72 has digits that add to 9, which is divisible by 3, and so:  $72=3 \times 24$

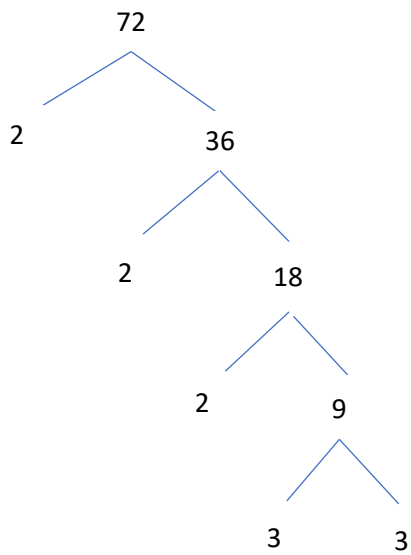
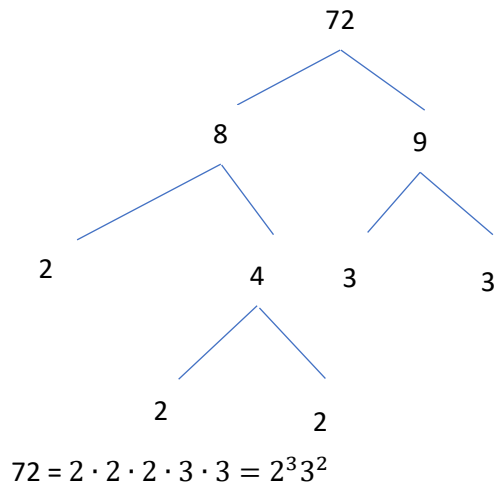
When I divided by 2, I got an even number, so I could divide by 2 again:  $4 \times 18$

It is divisible by 6 because I can divide separately by 2 and by 3:  $6 \times 12$

And we shouldn't forget:  $1 \times 72$

What are all the all the possible factors of 72? 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Another strategy to get to a set of factors is to get to a prime factoring and then recombine them to get the rest.



We still end up in the same place to get the prime factors.

To get the composite factors:

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$$

$$2 \cdot (2 \cdot 2 \cdot 3 \cdot 3) = 2 \times 36$$

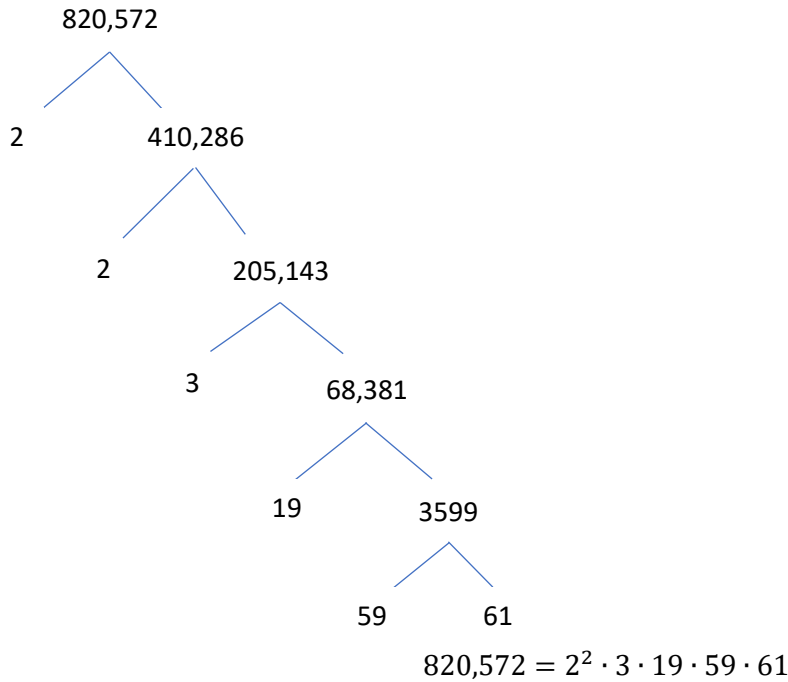
$$2 \cdot 2 \cdot (2 \cdot 3 \cdot 3) = 4 \times 18$$

$$2 \cdot 2 \cdot 2 \cdot (3 \cdot 3) = 8 \times 9$$

$$2 \cdot 2 \cdot 3 \cdot (3 \cdot 2) = 12 \times 6$$

$$(2 \cdot 2 \cdot 2 \cdot 3) \cdot 3 = 24 \times 3$$

Find the prime factors of 820,572



The rule is that you check primes until the result of your division is smaller than the number you divided by.

Or check primes until you reach  $\sqrt{\text{number you are dividing into}}$   
 $\sqrt{3599} \approx 59.99166 \dots$

If I've checked all the primes up to 59.99... then I can quit.

If you don't find any prime factors before this point, then the number is itself prime.

Least common multiple (LCM) vs. greatest common factor (GCF)

Least common multiple: find a number that both the number divide into evenly.

12: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, ...

20: 20, 40, 60, 80, 100, 120, 140, ...

There are several (many) common multiples that exist. (red)

We actually want the least common multiple, the smallest one. (yellow highlighter)

Greatest common factor is the largest number that divides into both numbers evenly.

Factors of the numbers:

12: 1, 2, 3, 4, 6, 12

20: 1, 2, 4, 5, 10, 20

The common factors are listed in red

The yellow highlight is the greatest common factor.

If the numbers are relatively prime, the only common factor is 1 (there this is the greatest common factor).

You can also get this from the prime factors:

$$12 = 2^2 \cdot 3$$

$$20 = 2^2 \cdot 5$$

If you have prime factors, the greatest common factor is any factor they have in common, with the smallest exponent between them for that prime factor.

$$12 = 2^2 \cdot 3$$

$$18 = 2 \cdot 3^2$$

$$GCF = 2 \cdot 3 = 6$$

Least common multiple with prime factors:

Take any factors they have in common and anything they don't have in common:

$$12 = 2^2 \cdot 3$$

$$20 = 2^2 \cdot 5$$

Common:  $2^2$ , what is different: 3, 5. Multiply them all together:

$$2^2 \cdot 3 \cdot 5 = 60$$

Multiples are always bigger than or equal to the number you start with.

Factors are smaller than or equal to the number you start with.

Multiplying and Dividing Fractions.

Multiplying:

$$\frac{3}{7} \times \frac{5}{11} = \frac{15}{77}$$

$$\frac{2}{3} \times \frac{5}{8} = \frac{10}{24} = \frac{5}{12}$$

$$\frac{2}{3} \times \frac{5}{8} = \frac{5}{12}$$

Division:

If you multiply a number by its reciprocal, then that is equivalent to dividing by the original number.

$$4 \div 3 = 4 \times \frac{1}{3}$$

Reciprocals of whole numbers (n) are like  $1/n$ . Reciprocals of fractions are just flipping the fractions.

$5/8$  becomes  $8/5$

$$\frac{3}{7} \div \frac{5}{11} = \frac{3}{7} \times \frac{11}{5} = \frac{33}{35}$$

Example.

$$\frac{4}{9} \div \frac{8}{15} = \frac{4}{9} \times \frac{15}{8} = \frac{5}{6}$$

Addition and Subtraction:

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

$$\frac{3}{7} - \frac{2}{7} = \frac{1}{7}$$

When the denominators are not equal, we have to make them equal before we can add or subtract.

$$\frac{1}{2} + \frac{1}{3} = \frac{1}{2} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{2}{2} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

These denominators (2, 3) are relatively prime. So we can do a little bit of a trick to get equivalent fractions to add:

The least common multiple is the product of the two denominators, the new first numerator multiplies by the second denominator, and the new second numerator multiplies by the first denominator.

$$\frac{1}{12} + \frac{1}{20} = \frac{1}{4 \times 3} \cdot \frac{5}{5} + \frac{1}{4 \times 5} \cdot \frac{3}{3} = \frac{5}{60} + \frac{3}{60} = \frac{8}{60} = \frac{2}{15}$$

What do I have to multiply 12 by to get to 60? What do I have to multiply 20 by to get to 60.

Least common denominator = least common multiple

LCD = LCM

If you have an exponent with a fraction:  $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$

If you have a root with a fraction:  $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$