6/16/2023

Mixed numerals
Operations
Proper vs. improper fractions

Fractions: ratio of integer divided another integer $\frac{p}{q}$

Proper fraction vs. improper fraction
Proper fraction: is a value between 0 and 1; the top number (numerator) is smaller than the bottom number (denominator)
Improper fraction: is a value is bigger than 1, the numerator is larger than the denominator

Proper fraction: $\frac{1}{3}, \frac{10}{11}, -\frac{5}{8}, etc.$

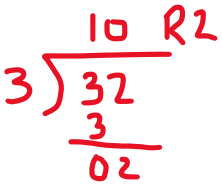
Improper fraction: $\frac{11}{10}, \frac{4}{3}, -\frac{126}{85}, etc.$

Mixed numbers: combinations of whole number and proper fractions: $4\frac{2}{3}$
These forms are useful for human interpretation of a result. Can be a useful form particularly at the end of a problem where we are sure we are done doing math.

Mathematics prefers to do operations on improper fractions (the math is just easier).

Converting improper fractions to mixed numerals:

$$\frac{128}{12} = \frac{64}{6} = \frac{32}{3}$$



First simplify, then do long division.

The mixed number has the result of the long division (main part) as the whole number, and the remainder is the numerator of the accompanying fraction with the original denominator.

$$\frac{32}{3} = 10\frac{2}{3}$$

Convert the mixed number into an improper fraction:

$$5\frac{3}{4} = 5 + \frac{3}{4} = \frac{5}{1} + \frac{3}{4} = \frac{5}{1} \cdot \frac{4}{4} + \frac{3}{4} = \frac{20}{4} + \frac{3}{4} = \frac{23}{4}$$

Mixed number form: $a\frac{b}{c}$, the improper fraction is $\frac{a \cdot c + b}{c}$

For our example: $\frac{5 \cdot 4 + 3}{4} = \frac{20+3}{4} = \frac{23}{4}$

Example.

Convert $7\frac{5}{9}$ to an improper fraction

$$7\frac{5}{9}=\frac{7\times 9+5}{9}=\frac{63+5}{9}=\frac{68}{9}$$

Operations on mixed numbers.

Distributive property.

$$a(b+c)=ab+ac$$

$$2\left(5\frac{3}{4}\right)=2\left(5+\frac{3}{4}\right)=2(5)+2\left(\frac{3}{4}\right)=10+\frac{\cancel{2}^{1}}{1}\times\frac{3}{\cancel{4}_{2}}=10+\frac{3}{2}=10+1\frac{1}{2}=11\frac{1}{2}$$

$$\left(2\frac{1}{2}\right)\left(5\frac{3}{4}\right)=\left(2\frac{1}{2}\right)\left(5+\frac{3}{4}\right)=\left(2\frac{1}{2}\right)(5)+\left(2\frac{1}{2}\right)\left(\frac{3}{4}\right)=\left(2+\frac{1}{2}\right)(5)+\left(2+\frac{1}{2}\right)\left(\frac{3}{4}\right)=$$

$$2(5)+\frac{1}{2}(5)+2\left(\frac{3}{4}\right)+\frac{1}{2}\left(\frac{3}{4}\right)=10+\frac{1}{2}\left(\frac{5}{1}\right)+\frac{2}{1}\left(\frac{3}{4}\right)+\frac{1}{2}\left(\frac{3}{4}\right)=10+\frac{5}{2}+\frac{3}{2}+\frac{3}{8}=$$

$$10+\frac{8}{2}+\frac{3}{8}=10+4+\frac{3}{8}=14\frac{3}{8}$$

We've seen two examples of working directly with mixed numbers (multiplication). There are a lot of steps!!!

Redo the problems using improper fractions instead.

$$2\left(5\frac{3}{4}\right)=2\left(\frac{23}{4}\right)=\frac{\cancel{2}^{1}}{1}\left(\frac{23}{\cancel{4}_{2}}\right)=\frac{23}{2}=11\frac{1}{2}$$

$$\left(2\frac{1}{2}\right)\left(5\frac{3}{4}\right)=\left(\frac{5}{2}\right)\left(\frac{23}{4}\right)=\frac{115}{8}=14\frac{3}{8}$$

$$2\overline{)23}\ 11\ R1,\quad 2,\quad 03,\quad 2,\quad 1$$

$$8\overline{)115}\ 14\ R3,\quad 8,\quad 35,\quad 32,\quad 3$$

Division.

$$\frac{7\frac{2}{9}}{2\frac{1}{3}}=\left(7\frac{2}{9}\right)\div\left(2\frac{1}{3}\right)=\left(\frac{65}{9}\right)\div\left(\frac{7}{3}\right)=\frac{65}{\cancel{9}_{3}}\times\frac{\cancel{3}^{1}}{7}=\frac{65}{21}=3\frac{2}{21}$$

$$21\overline{)65}\ 3\ R2,\quad 63,\quad 2$$

Addition.

Adding mixed numbers with a like denominator

$$3\frac{1}{5}+4\frac{3}{5}=3+\frac{1}{5}+4+\frac{3}{5}=(3+4)+\left(\frac{1}{5}+\frac{3}{5}\right)=7+\frac{4}{5}=7\frac{4}{5}$$

Add mixed numbers with unlike denominators

$$4\frac{2}{7}+5\frac{10}{21}=(4+5)+\left(\frac{2}{7}+\frac{10}{21}\right)=9+\left(\frac{2}{7}\cdot\frac{3}{3}+\frac{10}{21}\right)=9+\frac{6}{21}+\frac{10}{21}=9+\frac{16}{21}=9\frac{16}{21}$$

Adding with improper fractions:

$$3\frac{1}{5}+4\frac{3}{5}=\frac{16}{5}+\frac{23}{5}=\frac{39}{5}=7\frac{4}{5}$$

$$4\frac{6}{7}+5\frac{10}{21}=\frac{34}{7}+\frac{115}{21}=\frac{34}{7}\cdot\frac{3}{3}+\frac{115}{21}=\frac{102}{21}+\frac{115}{21}=\frac{217}{21}=10\frac{7}{21}=10\frac{1}{3}$$

Subtracting with mixed numbers:

$$5\frac{1}{3}-2\frac{2}{3}=$$

~~5~~ 4 1/3
- 2 2/3
2 2/3

Subtract with improper fractions

$$5\frac{1}{3}-2\frac{2}{3}=\frac{16}{3}-\frac{8}{3}=\frac{8}{3}=2\frac{2}{3}$$

$$8\frac{1}{7}-4\frac{8}{9}=\frac{57}{7}-\frac{44}{9}=\frac{57}{7}\cdot\frac{9}{9}-\frac{44}{9}\cdot\frac{7}{7}=\frac{513}{63}-\frac{308}{63}=\frac{205}{63}=3\frac{16}{63}$$

Exponents.

$$\left(5\frac{3}{4}\right)^3=\left(\frac{23}{4}\right)^3=\frac{12167}{64}=190\frac{7}{64}$$

Roots.

$$\sqrt{5\frac{4}{9}} = \sqrt{\frac{49}{9}} = \frac{\sqrt{49}}{\sqrt{9}} = \frac{7}{3} = 2\frac{1}{3}$$

Order of operations remains the same.
You may find it helpful to put mixed number in parentheses until you convert them to improper fractions.

Next up: decimals.

Aside: Division by 0

$$7 \times 8 = 56$$

$$\frac{56}{7} = 8$$

$$\frac{56}{8} = 7$$

$$0 \times 0 = 0$$

$$\frac{0}{0} = 0$$

But:

$$0 \times 125 = 0$$

$$\frac{0}{125} = 0$$

$$\frac{0}{0} = 125$$

Dividing 0/0 has more than one answer.

$$\frac{3}{0} = ??? \, (undefined)$$

$$a \times 0 = 3$$

No such number exists because any number multiplied by 0 must be zero, and can't be 3.

Negative exponents are division:

$$7 \times 7 = 7^2$$

$$7^{-2} = \frac{1}{7} \times \frac{1}{7}$$