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Order of operations
Exponents (rational/roots)
Evaluating expressions

Typical way order of operations is taught in American math courses is using the mnemonic device PEMDAS (Please Excuse My Dear Aunt Sally).

P – parentheses
E – exponents (and roots)
MD – multiplication and division: these are done together left to right
AS – addition and subtraction: these are done together left to right

It is possible to do order of operations in six steps with one operation per step:
P – parentheses
E – exponents
D – division
M – multiplication
S – subtraction
A – addition

PEDMSA

Examples.

$$9 \div 3 + 2(9 + 10) - 8 + 4 \times 3$$

Method 1: PEMDAS

$$9 \div 3 + 2(9 + 10) - 8 + 4 \times 3$$
$$9 \div 3 + 2(19) - 8 + 4 \times 3$$
$$3 + 2(19) - 8 + 4 \times 3$$
$$3 + 38 - 8 + 4 \times 3$$
$$3 + 38 - 8 + 12$$
$$41 - 8 + 12$$
$$33 + 12$$
$$45$$

Method 2: PEDMSA

$$9 \div 3 + 2(9 + 10) - 8 + 4 \times 3$$
$$9 \div 3 + 2(19) - 8 + 4 \times 3$$
$$3 + 2(19) - 8 + 4 \times 3$$
$$3 + 2(19) - 8 + 12$$
$$3 + 38 - 8 + 12$$
$$3 + 30 + 12$$
$$33 + 12$$
$$45$$

$$4 \times 5 \div 2$$

In this example, I would do the multiplication first to get $20 \div 2 = 10$ rather than do $\frac{5}{2}$ first because this doesn't divide evenly.

But if I had $4 \times 6 \div 3$ then I would do $6 \div 3 = 2$ first, then multiply by 4 to 8

Exponents and roots
There is way to express roots as exponents, as rational exponents

$$\sqrt{a} = a^{1/2}$$

$$\sqrt[3]{a} = a^{1/3}$$

$$\sqrt[4]{a} = a^{1/4}$$

$$\sqrt{2^2} = (2^2)^{1/2} = 2^{2\left(\frac{1}{2}\right)} = 2^1 = 2$$

$$\sqrt[3]{4^3} = (4^3)^{\frac{1}{3}} = 4^{3\left(\frac{1}{3}\right)} = 4^1 = 4$$

$$\sqrt{4^8} = \left(\sqrt{4}\right)^8 = 2^8 = 256$$

$$\sqrt{4^8} = (4^8)^{1/2} = 4^{8\left(\frac{1}{2}\right)} = 4^4 = 256$$

As opposed to $\sqrt{4^8} = \sqrt{65{,}536}$

$$\frac{1}{\sqrt{a}} = a^{-1/2}$$

The numerator of the exponent is the power, and the denominator is the root

$$a^{3/5} = \sqrt[5]{a^3}$$

$$32^{3/5} = \sqrt[5]{32^3} = \sqrt[5]{32768} = \left(\sqrt[5]{32}\right)^3 = 2^3 = 8$$

You can use the rational exponents in the calculator to get alternate roots.

Note about calculators and scientific notation:

The calculator will write 1×10^{-6} as 1e-6 or 1E-6. The "e" or "E" is standing in for "$\times 10$ " and the number that follows it is the power.

But also don't write that as an answer to a problem. You need to write your answers in standard scientific notation with the powers of 10, not e.

Simplifying roots.

$$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2 \times \sqrt{3} = 2\sqrt{3}$$

$$\frac{\sqrt{18}}{\sqrt{6}} = \sqrt{\frac{18}{6}} = \sqrt{3} = \frac{\sqrt{9 \times 2}}{\sqrt{3 \times 2}} = \frac{\sqrt{9} \times \sqrt{2}}{\sqrt{3} \times \sqrt{2}} = \frac{\sqrt{9}}{\sqrt{3}} = \frac{3}{\sqrt{3}}$$

Evaluating Expressions (formulas)
Algebraic expression is a "statement" with variables in it
Evaluation means we are replacing those variables with numbers and then simplifying to get the result.

$$3x + 5$$
$$\frac{a^3}{b^7 c^4}$$

Evaluate the expression x^2 when $x = 3.2$

$$3.2^2 = 10.24$$

Evaluate the expression $x^3 - 2x^2 + x - 11$ when $x = -1$

$$(-1)^3 - 2(-1)^2 + (-1) - 11 = (-1) - 2(1) + (-1) - 11 = -1 - 2 - 1 - 11 = -15$$

Evaluate the expression $\frac{5}{9}(F-32)$ when $F=158$

$$\frac{5}{9}(158-32)=\frac{5}{9}(126)=5(14)=70$$

Evaluate the expression $\frac{a^3b^4}{c^2d^5}$ when $a=4, b=-3, c=6, d=-2$

$$\frac{4^3(-3)^4}{6^2(-2)^5}=\frac{(64)(81)}{(36)(-32)}=\frac{2(9)}{(4)(-1)}=\frac{9}{-2}=-4.5$$