MTH 263, Homework #2, Summer 2023 Name KEY

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Find the derivative using the limit definition.
   a. $f(x) = -3$
   b. $h(x) = 2 - x^2$
   c. $s(x) = \frac{1}{x^2}$
   d. $g(x) = 8 - \frac{1}{5}x$
   e. $q(x) = x^3 + x^2$
   f. $t(x) = \sqrt{x+4}$

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2. Find the equation of the tangent line at the given point.
   a. $f(x) = \frac{3}{2}x + 1, (-2,-2)$
   b. $h(t) = t^2 + 3, (-2,7)$

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3. Use differentiation rules to find the derivatives.
   a. $y = x^{16}$
   b. $s(t) = t^3 + 5t^2 - 3t + 8$
   c. $f(t) = 3 - \frac{3}{5t}$
   d. $g(x) = \sqrt[6]{x}$
   e. $h(t) = t^3 + 2e^t$
   f. $q(x) = \frac{x^3+3x^2+4}{x^2}$

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4. Determine the points, if any, at which the function has a horizontal tangent line.
   a. $y = x^4 - 2x^2 + 3$
   b. $y = -4x + e^x$

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5. Find the average rate of change on the interval.
   a. $f(x) = -\frac{1}{x}, [1,2]$
   b. $g(x) = x^2 + e^x, [0,1]$

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6. Find the derivative.
   a. $f(x) = xe^x$
   b. $h(x) = \sqrt{x}(x^2 + 8)$
   c. $m(s) = \frac{s}{\sqrt{s}-1}$
   d. $g(x) = (x^2 + 3)(x^2 - 4x)$
   e. $q(x) = \frac{x}{x^2+1}$
   f. $s(x) = x^4\left(1 - \frac{2}{x+1}\right)$

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7. Find the second and third derivatives of the function.
   a. $f(x) = 4x^{\frac{3}{2}}$
   b. $h(x) = \frac{e^x}{x}$
   c. $g(x) = \frac{x^2+2x-1}{x}$

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1a. $\lim_{h\to 0} \frac{-3-(-3)}{h} = 0$

b. $\lim_{h\to 0} \frac{2-(x+h)^2-(2-x^2)}{h} = \lim_{h\to 0} \frac{\cancel{2}-\cancel{x^2}-2xh-h^2-\cancel{2}+\cancel{x^2}}{h} = \lim_{h\to 0} \frac{-2xh-h^2}{h} =$

$\lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}} = \lim_{h\to 0} -2x-h = -2x$

1c. $\lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{x^2-(x+h)^2}{x^2(x+h)^2}\right) = \lim_{h\to 0} \frac{1}{h}\left(\frac{\cancel{x^2}-\cancel{x^2}-2xh-h^2}{x^2(x+h)^2}\right) = \lim_{h\to 0} \frac{-2x-h}{x^2(x+h)^2} =$

$\frac{-2x}{x^4} = -\frac{2}{x^3}$

d. $\lim_{h\to 0} \frac{8-\frac{1}{5}(x+h)-(8-\frac{1}{5}x)}{h} = \lim_{h\to 0} \frac{\cancel{8}-\cancel{\frac{1}{5}x}-\frac{1}{5}h-\cancel{8}+\cancel{\frac{1}{5}x}}{h} = \lim_{h\to 0} \frac{-1/5\cancel{h}}{\cancel{h}} = -\frac{1}{5}$

e. $\lim_{h\to 0} \frac{(x+h)^3+(x+h)^2-(x^3+x^2)}{h} = \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3+\cancel{x^2}+2xh+h^2-\cancel{x^3}-\cancel{x^2}}{h} =$

$\lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2)+\cancel{h}(2x+h)}{\cancel{h}} = \lim_{h\to 0} 3x^2+3xh+h^2+2x+h = 3x^2+2x$

f. $\lim_{h\to 0} \frac{\sqrt{x+h+4}-\sqrt{x+4}}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h+4}-\sqrt{x+4})(\sqrt{x+h+4}+\sqrt{x+4})}{h(\sqrt{x+h+4}+\sqrt{x+4})} = \lim_{h\to 0} \frac{x+h+4-(x+4)}{h(\sqrt{x+h+4}+\sqrt{x+4})} =$

$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+4}+\sqrt{x+4})} = \frac{1}{\sqrt{x+4}+\sqrt{x+4}} = \frac{1}{2\sqrt{x+4}}$

2a. $f(x) = \frac{3}{2}x+1 \to f'(x) = \frac{3}{2}$

$y+2 = \frac{3}{2}(x+2)$

b. $h(t) = t^2+3,\quad h'(t) = 2t$

$h'(-2) = -4$

$y-7 = -4(x+2)$

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3a. $y' = 16x^{15}$

b. $s'(t) = 3t^2 + 10t - 3$

c. $f'(t) = -\frac{3}{5}(-t^{-2}) = \frac{3}{5t^2}$

d. $g'(x) = \frac{1}{6}x^{-5/6} = \frac{1}{6\sqrt[6]{x^5}}$

e. $h'(t) = 3t^2 + 2e^t$

f. $g'(x) = 1 - 8x^{-3} = 1 - \frac{8}{x^3}$ $\qquad x + 3 + 4x^{-2}$

4a. $y' = 4x^3 - 4x$

$4x(x^2-1)$ $\qquad x = 0, 1, -1$

$4x(x-1)(x+1) = 0$

b. $y' = -4 + e^x = 0$

$e^x = 4$

$x = \ln 4$

5a. $f(1) = -\frac{1}{1} = -1$ $\qquad m = \frac{-\frac{1}{2} - (-1)}{2-1} = \frac{-\frac{1}{2}+1}{1} = \frac{1}{2}$

$f(2) = -\frac{1}{2}$

b. $f(0) = 0 + 1 = 1$ $\qquad m = \frac{1+e-1}{1-0} = e$

$f(1) = 1 + e$

6a. $f'(x) = e^x + xe^x$

b. $h'(x) = \frac{1}{2\sqrt{x}}(x^2+8) + \sqrt{x}(2x)$

c. $m'(s) = \frac{1(\sqrt{s}-1) - \frac{1}{2\sqrt{s}}(s)}{(\sqrt{s}-1)^2}$

f. $S'(x) = 4x^3\left(1 - \frac{2}{x+1}\right) + x^4\left(\frac{2}{(x+1)^2}\right)$

d. $g'(x) = 2x(x^2-4x) + (x^2+3)(2x-4)$

e. $g'(x) = \frac{1(x^2+1) - 2x(x)}{(x^2+1)}$

7a. $f(x) = 4x^{3/2}$

$f'(x) = 4\left(\frac{3}{2}\right)x^{1/2} = 6x^{1/2}$

$f''(x) = 6 \cdot \frac{1}{2}x^{-1/2} = \frac{3}{\sqrt{x}}$

$f'''(x) = 3\left(-\frac{1}{2}\right)x^{-3/2} = -\frac{3}{2\sqrt{x^3}}$

b. $h(x) = \frac{e^x}{x}$

$h'(x) = \frac{e^x(x) - e^x}{x^2} = \frac{e^x(x-1)}{x^2}$

$h''(x) = e^x\left(\frac{1x^2 - 2x(x-1)}{x^4}\right) + e^x\left(\frac{x-1}{x^2}\right) = e^x\left(\frac{x^2 - 2x^2 + 2x}{x^4} + \frac{x-1}{x^2}\right)$

$= e^x\left(\frac{-x^2+2x}{x^4} + \frac{x^3 - x^2}{x^4}\right) = e^x\left(\frac{-2x^2+2x+x^3}{x^4}\right) = e^x\left(\frac{x^2-2x+2}{x^3}\right)$

$e^x(x^{-1} - 2x^{-2} + 2x^{-3})$

$h'''(x) = e^x(x^{-1} - 2x^{-2} + 2x^{-3}) + e^x(-x^{-2} + 4x^{-3} - 6x^{-4})$

$= e^x(x^{-1} - 2x^{-2} + 2x^{-3} - x^{-2} + 4x^{-3} - 6x^{-4}) =$

$e^x(x^{-1} - 3x^{-2} + 6x^{-3} - 6x^{-4}) = e^x\left(\frac{x^3 - 3x^2 + 6x - 6}{x^4}\right)$

c. $g(x) = \frac{x^2+2x-1}{x} = x + 2 - x^{-1}$

$g'(x) = 1 + x^{-2} = 1 + \frac{1}{x^2}$

$g''(x) = -2x^{-3} = -\frac{2}{x^3}$

$g'''(x) = 6x^{-4} = \frac{6}{x^4}$