MTH 263, Homework #5, Summer 2023 Name KEY

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Sketch the graph of the function by hand. Label any intercepts, relative extrema, points of inflection and asymptotes. You may use a calculator to confirm your results.
   a. $y = \frac{x^2}{x^2+3}$
   b. $y = x\sqrt{x+4}$
   c. $y = x^4 - 4x^3 + 16x$
   d. $y = (5-x)5^x$
   e. $y = x + \frac{32}{x^2}$
   f. $y = 3x^{2/3} - 2x$
   g. $y = (x-1)\ln(x-1)$

   pg 2-3

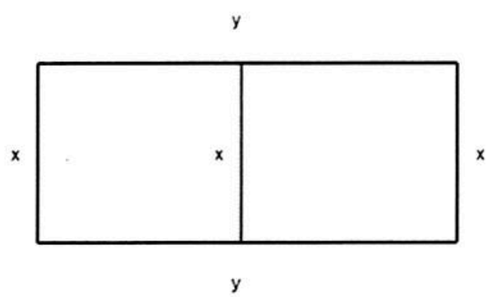
2. Find the limit if it exists.
   a. $\lim\limits_{x\to\infty} \frac{x^2+2}{x^3-1}$
   b. $\lim\limits_{x\to-\infty} \frac{\sqrt{x^4-1}}{x^3-1}$
   c. $\lim\limits_{x\to\infty} \frac{5-2x^{3/2}}{3x^{3/2}-4}$
   d. $\lim\limits_{x\to\infty} \cos\left(\frac{1}{x}\right)$

   pg 3

3. Find 2 positive numbers such that the sum is 5 and the product is a maximum. pg 3

4. Find the length and width of a rectangle with an area of 32 ft^2 with the minimum perimeter. pg 3

5. A rancher has 400 feet of fencing with which to enclose 2 adjacent rectangular corrals. What dimensions should be used so that the area is a maximum.



pg 3

6. A rectangular package sent through the postal service can have a maximum combined length (ℓ) and girth (hw) of 108 inches. Find the maximum volume of a package that can be sent.

7. Use L'Hôpital's Rule to find the limit.
   a. $\lim\limits_{x\to0} \frac{\sin 6x}{4x}$
   b. $\lim\limits_{x\to1} \frac{\ln x^2}{x^2-1}$
   c. $\lim\limits_{x\to\infty} \frac{x^3}{e^{x/2}}$
   d. $\lim\limits_{x\to0} \frac{x}{\arctan 2x}$
   e. $\lim\limits_{x\to1^+} (\ln x)^{x-1}$

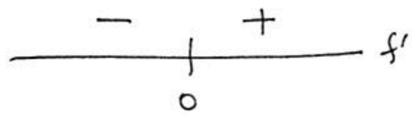
   pg 3

   g. $\lim\limits_{x\to\infty} \frac{5x^2-3x+1}{3x^2-5}$
   h. $\lim\limits_{x\to0^+} \frac{e^x-(1+x)}{x^3}$
   i. $\lim\limits_{x\to\infty} \frac{\ln x^4}{x^3}$
   j. $\lim\limits_{x\to\infty} x\sin\left(\frac{1}{x}\right)$
   k. $\lim\limits_{x\to\infty} (1+x)^{1/x}$

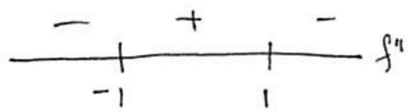
   pg 4

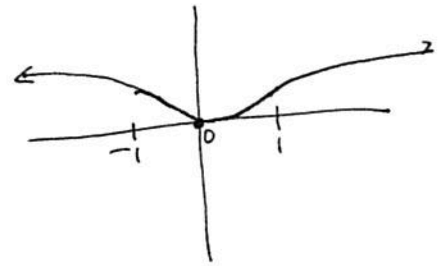
MTH 263 Homework #5 Key

1a. $y' = \dfrac{6x}{(x^2+3)^2}$ $x=0$



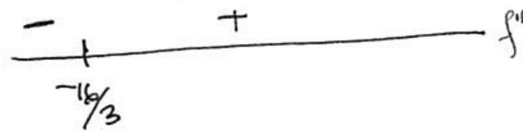
$y'' = \dfrac{-18(x^2-1)}{(x^2+3)^3}$ $x=\pm 1$

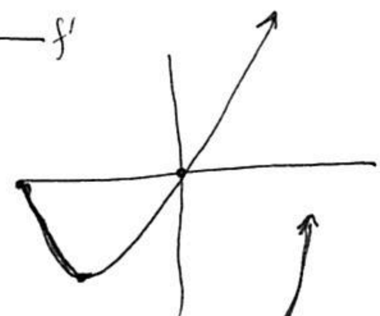




1b. $y' = \dfrac{3x+8}{2\sqrt{x+4}}$ $x = \dfrac{-8}{3}$

$y'' = \dfrac{3x+16}{4(x+4)^{3/2}}$ $x = \dfrac{-16}{3}$



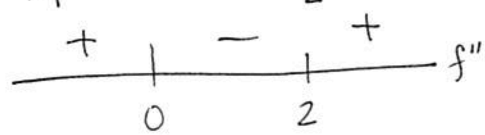


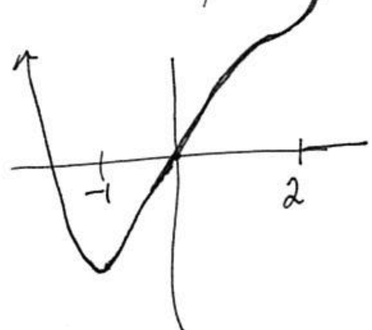
c. $y' = 4x^3-12x^2+16$ $4(x^3-3x^2+4)$ $x=2,-1$

$y'' = 12x^2-24x$

$12x(x-2)$

$x=0,\ x=2$





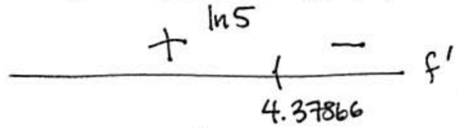
d. $y' = -5^x(\ln 5 \cdot x + 1 - 5\ln 5)$ $\ln 5 \cdot x = 5\ln 5 - 1$

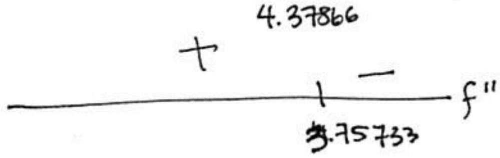
$x = \dfrac{5\ln 5 - 1}{\ln 5} \approx 4.37866\ldots$

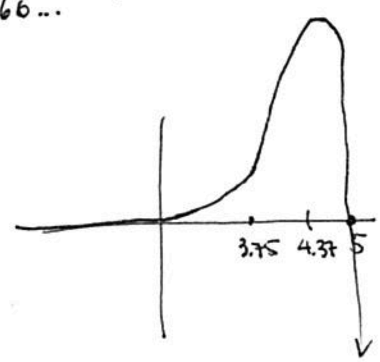
$y'' = -5^x(\ln 5)(x \cdot \ln 5 + 2 - 5\ln 5)$

$x \cdot \ln 5 = 5\ln 5 - 2$

$x = \dfrac{5\ln 5 - 2}{\ln 5} \approx 3.75733\ldots$



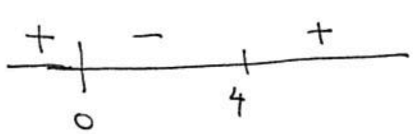




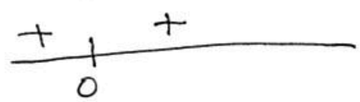
e. $y' = 1-64x^{-3}$ $\dfrac{64}{x^3} = 1 \Rightarrow x^3 = 64$

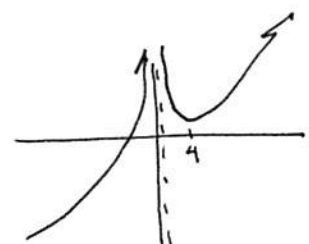
$x=4$

$x=0$



$y'' = \dfrac{256}{x^4}$

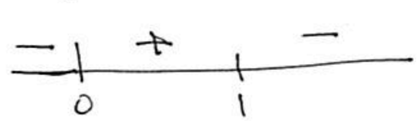




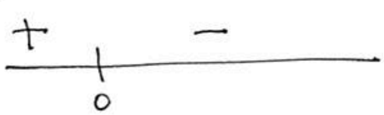
f. $y' = 2x^{-1/3} - 2 = 0$ $2 = 2x^{-1/3}$

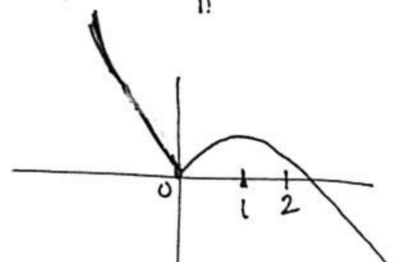
$x^{1/3} = 1$ $x=1$

$x=0$



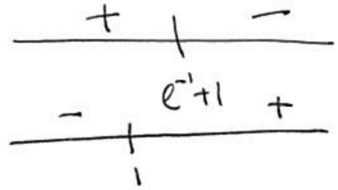
$y'' = -\frac{2}{3}x^{-4/3} = 0$ $x=0$

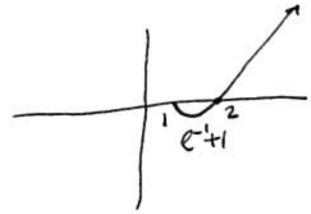




1g. $y' = \ln(x-1)+1$ $\quad \ln(x-1) = -1$
$\quad x-1 = e^{-1}$
$\quad x = e^{-1}+1$

$y'' = \dfrac{1}{x-1} \quad x=1$





2a. $\lim\limits_{x\to\infty} \dfrac{x^2+2}{x^3-1} = \lim\limits_{x\to\infty} \dfrac{2x}{3x^2} = \lim\limits_{x\to\infty} \dfrac{2}{6x} = 0$

b. $\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^4-1}}{x^3-1} = 0$

c. $\lim\limits_{x\to\infty} -\dfrac{2}{3} \qquad \lim\limits_{x\to\infty} \cos\left(\dfrac{1}{x}\right) = 1$

3. $x+y=5 \to y = 5-x$
$xy = p$
$x(5-x) = p \Rightarrow 5x - x^2 = p \quad p'(x) = 5-2x = 0 \quad 2x=5 \Rightarrow x = 5/2$
$x = y = 2.5$

4. $lw = 32 \to l = \dfrac{32}{w}$
$P = 2l+2w \to P = 2\left(\dfrac{32}{w}\right)+2w \to P = \dfrac{64}{w}+2w \quad P' = \dfrac{-64}{x^2}+2=0 \quad 2 = \dfrac{64}{x^2} \Rightarrow x^2 = 32$
$x = 4\sqrt{2}$
$l = w = 4\sqrt{2}$ max

5. $P = 3x+2y \Rightarrow 400 = 3x+2y \to 400-3x = 2y \to 200 - \frac{3}{2}x = y$
$A = xy \Rightarrow A = x(200-\frac{3}{2}x) = 200x - \frac{3}{2}x^2$
$A' = 200 - 3x = 0 \quad x = \dfrac{200}{3} \quad y = 200 - \frac{3}{2}\left(\dfrac{200}{3}\right) = 200-100 = 100$

6. $l + hw = 108 \to hw = 108 - l$
$V = lwh \to V = l(108-l) = 108l - l^2 \quad V' = 108-2l = 0 \quad 54 = l$
$hw = 54 \quad V = 54^2 = 2916$ if $h = w = 14.2866$

7a. $\lim\limits_{x\to 0} \dfrac{\sin 6x}{4x} = \dfrac{6}{4} = \dfrac{3}{2}$

b. $\lim\limits_{x\to 1} \dfrac{\ln x^2}{x^2-1} = \lim\limits_{x\to 1} \dfrac{2\ln x}{x^2-1} = \lim\limits_{x\to 1} \dfrac{2/x}{2x} = \lim\limits_{x\to 1} \dfrac{1}{x^2} = 1$

c. $\lim\limits_{x\to\infty} \dfrac{x^3}{e^{x/2}} = \lim\limits_{x\to\infty} \dfrac{3x^2}{\frac{1}{2}e^{x/2}} = \lim\limits_{x\to\infty} \dfrac{6x}{\frac{1}{4}e^{x/2}} = \lim\limits_{x\to\infty} \dfrac{6}{\frac{1}{8}e^{x/2}} = 0$

d. $\lim\limits_{x\to 0} \dfrac{x}{\arctan 2x} = \lim\limits_{x\to 0} \dfrac{1}{\frac{2}{1+4x^2}} = \lim\limits_{x\to 0} \dfrac{1+4x^2}{2} = \dfrac{1}{2}$

e. $\lim\limits_{x\to 1^+} (\ln x)^{x-1} \to \lim\limits_{x\to 1^+} (x-1)\ln(\ln x) = \lim\limits_{x\to 1^+} \dfrac{\ln(\ln x)}{(x-1)^{-1}} = \lim\limits_{x\to 1^+} \dfrac{\frac{1}{\ln x \cdot x}}{-(x-1)^{-2}} = \lim\limits_{x\to 1^+} \dfrac{-(x-1)^2}{x\ln x} = \lim\limits_{x\to 1^+} \dfrac{-2(x-1)}{\ln x + 1}$
$= 0 \longrightarrow 0 = \ln L \to L = 1$

7. there is no f.

g. $$\lim_{x\to\infty} \frac{5x^2-3x+1}{3x^2-5} = \frac{5}{3}$$

h. $$\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3} = \lim_{x\to 0^+} \frac{e^x-1}{3x^2} = \lim_{x\to 0^+} \frac{e^x}{6x} = \infty$$

i. $$\lim_{x\to\infty} \frac{\ln x^4}{x^3} = \lim_{x\to\infty} \frac{4\ln x}{x^3} = \lim_{x\to\infty} \frac{4/x}{3x^2} = \lim_{x\to\infty} \frac{4}{3x^3} = 0$$

j. $$\lim_{x\to\infty} x\sin\left(\tfrac{1}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{1/x} = \lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)\cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = 1$$

k. $$\lim_{x\to\infty} (1+x)^{1/x} \Rightarrow \lim_{x\to\infty} \frac{1}{x}\ln(1+x) = \lim_{x\to\infty} \frac{\ln(1+x)}{x} = \lim_{x\to\infty} \frac{1/(1+x)}{1} = \lim_{x\to\infty} \frac{1}{1+x} = 0$$

$$0 = \ln L \Rightarrow L = 1$$